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Research on D-H Parameter Modeling Methods

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Abstract

A robot manipulator consists of several links connected by usually of single degree of freedom joints say, a revolute or a prismatic joint. In order to control the end-effector with respect to the base, it's necessary to find a relation between the end-effector and the base. The D-H parameter modeling method is the most popular due to the simplicity and validity. This paper studies two different D-H parameter modeling methods. The D-H parameter was proposed by Denavit and Hartenberg to represent a directed the axis line of a lower pair joint. However, in the subsequent application process, Paul and Craig improved them one after another to facilitate calculation and memory, which are respectively called Standard D-H parameter modeling method and Modified D-H parameter modeling method. However, some literatures show that the difference between the two methods in practical application is somewhat confusing. For example, someone attaches the coordinate frames through Standard DH modeling method, but calculate through the homogeneous transformation matrix of the Modified D-H parameter modeling method, which makes wrong model. Since most robotic mechanisms are essentially designed for motion, the kinematic modeling of a robot manipulator is very important that describes the relationship between the links and joints. This paper studies two different D-H parameter modeling methods. Both methods are here presented and compared and the tips to distinguish them are provided. Finally, the simulation of Staubil TX60L is carried out by using two methods on MATLAB.

Keywords

Standard Denavit-Hartenberg Parameter, Modified Denavit-Hartenberg Parameter, Robot, Simulation

1. Introduction

Over the past decades, industrial robots have become more and more important for manufacturing industry with the rising labor costs in China. In order to realize intelligent manufacturing, we must vigorously develop intelligent equipment like industrial robots which are called "Pearl at the Top of the Crown of Manufacturing Industry" [1, 2]. Since the first ISO-compliant industrial robot came out in 1938, the manipulator has gradually become one of the fastest-growing and most widely used industrial robots. A robot manipulator consists of several links connected by usually of single degree of freedom joints say, a revolute or a prismatic joint [3]. In order to design, control and study the manipulator, it is necessary to find the motion relationship between the end-effector and the base and establish its motion model. Because of its simplicity and effectiveness, one of the most popular modeling methods is the D-H parameter modeling method, which is the object of this study.

The D-H parameter was proposed by Denavit and Hartenberg to represent a directed the axis line of a lower pair joint [4] at first. Waldron [5] and Paul [6] revised the label of the joint axis in Denavit and Hartenberg's original rule in 1973 and 1982 respectively to unify the label of links and joint axis. The coordinate frame is attached to the joint axis, and the relationship between the joints and links is expressed by describing the relationship between the coordinate frames. This modeling method is called Standard D-H modeling method. Laterly Craig [7] changed the position of link coordinate frames. Generally, Craig's modified modeling method is called Modified D-H parameter modeling method.

The two methods look similar, which makes users confused. In fact, the different position of link coordinate frame causes the different order of link coordinate frame transformation and the different homogeneous transformation matrix. After reading an amount of literature, it's found that many users mix the two methods up. For example, the order of frame transformation is that rotate around Z, translate along Z, along X, rotate around X in literature [8] according to Standard D-H modeling method. But Modified DH modeling method is used to establish the coordinate frame of the robot afterwards. A similar situation occurred in [9] and [10]. Sometimes due to the particularity of some parameters of the manipulator, correct conclusions can be drawn even though the model is misused. In order to be academically rigorous, we must correctly understand and distinguish the two methods.

2. D-H Parameter

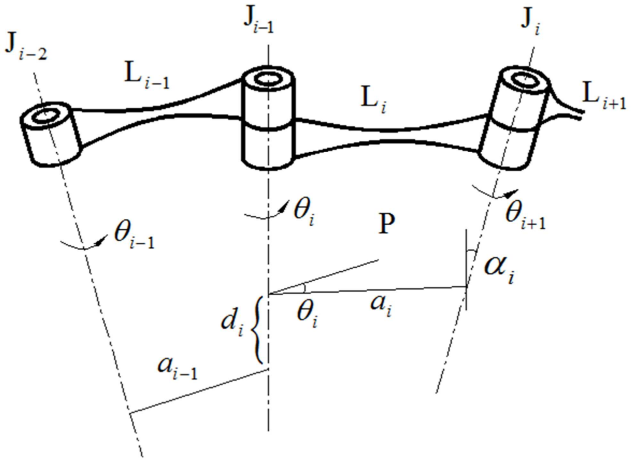

Figure 1. *D-H parameter original rule.*

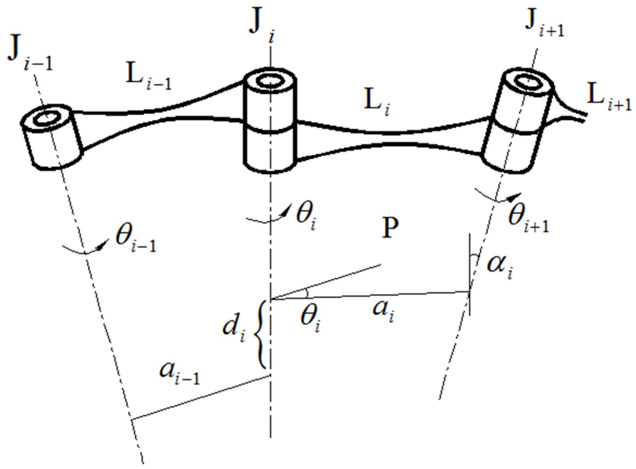

Figure 2. *DH parameter revision.*

In order to describe the pose relationship between the links of the manipulators, in 1955, Denavit and Hartenberg established a Cartesian coordinate system on each link in accordance with specific rules, and proposed the concept of the D-H parameter model, which only needs four parameters to represent the conversion relationship between two Cartesian coordinate systems. The original rule is shown in Figure 1, where J_i is between L_i and L_{i+1}, that is, the i joint is outside the L_i link, which causes the transformations d_i and θ_i to be along or around J_{i-1} instead of J_i. It is error-prone and inconvenient to remember when calculating. Waldon and Paul revised the joint axis marks in the original rules by Denavit and Hartenberg respectively in 1973 and 1982, which placed J_i between L_{i-1} and L_i, that is, the joint i is inside the link L_i, as shown in Figure 2. The improved rules ensures that the joint variable subscript is consistent with the joint axis subscript.

Due to convenience and uniformity, most scholars now choose the D-H parameters revised by Paul. The parameters are defined as follows:

(1) The length a_i of the links: the common perpendicular distance of the joint axes J_i and J_{i+1} of two neighboring joints, recorded as a_i;

(2) The torsion angle α_i of the links: the axis J_i and the above-mentioned common perpendicular form a plane P, the angle of the axis J_{i+1} and the plane P, is recorded as α_i;

(3) The offset d_i of the links: except the first link and the last link, there is always a common perpendicular a_i between the joint axes J_i and J_{i+1} of any two adjacent links, and the distance of two adjacent common perpendicular, a_{i-1} and a_i, of one joint, recorded as d_i;

(4) Joint angle θ_i: The angle of the projections of the adjacent two common perpendicular lines of the joint J_i on the plane with J_i as the normal, is recorded as θ_i.

3. Coordinate System of the Links

There are many ways to establish rectangular coordinate system on the links. Generally, it is customary to take the direction of axis Z parallel to axis J, and the direction of axis X parallel to the direction of common perpendicular of two axes. Therefore, in DH parameters, θ_i and d_i are transformed along or around axis Z, and a_i and α_i are transformed along or around axis X [11-13].

3.1. The Position of Coordinate System Setting

3.1.1. Standard D-H Model

After proposing the revised D-H parameters, Paul established link coordinate system in the way shown in Figure 3, which is called the Standard D-H parameter modeling method or the Classical D-H parameter modeling method.

The steps to establish the coordinate system are as follows:

(1) Determination of the origin O_i: take the intersection of the common vertical line of J_i and J_{i+1} at J_{i+1} as the origin of the coordinate system.

(2) Determination of the direction of the axis Z_i: take the direction of J_{i+1} as the direction of the axis Z_i;

(3) Determination of the direction of the axis X_i: take the

direction of J_i and J_{i+1} common vertical line pointing to O_i as the axis X_i direction.

(4) Determine the axis Y_i direction according to the right-hand rule.

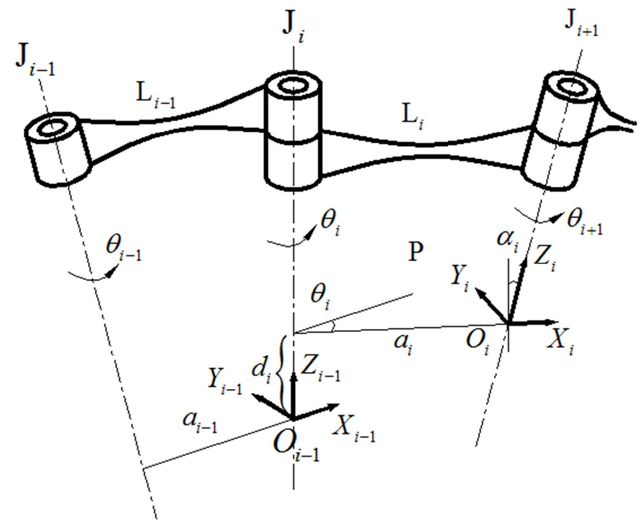

Figure 3. *Standard D-H model.*

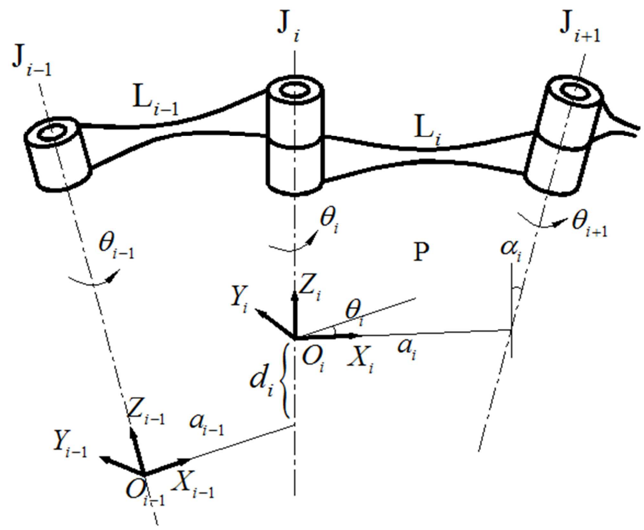

Figure 4. *Modified D-H model.*

3.1.2. Modified D-H Model

Paul configures the axis Z_i on J_{i+1}, which causes the subscript of i-link coordinate system not correspond to the joint axis subscript. So Craig proposed an revised D-H modeling method called Modified D-H modeling method in 1986, which configures the axis Z_i on J_i, as shown in Figure 4.

The steps to establish the coordinate system are as follows:

(1) Determination of the origin O_i: take the intersection of the common vertical line of J_i and J_{i+1} at J_i as the origin of the coordinate system.

(2) Determination of the direction of the axis Z_i: take the direction of J_i as the direction of the axis Z_i;

(3) Determination of the direction of the axis X_i: take the direction of J_i and J_{i+1} common vertical line from J_{i+1} pointing to O_i as the axis X_i direction.

(4) Determine the axis Y_i direction according to the right-hand rule.

3.2. Coordinate System Transformation Order

3.2.1. Standard D-H Parameter Modeling Method

For the link coordinate system established by Standard D-H parameter modeling method above, the link coordinate system L_{i-1} coincides with the link coordinate system L_i after two rotations and two translations. See Figure 3 and Figure 5.

The four transformations are as follows:

(1) Rotate θ_i-degree around the axis Z_{i-1}, so that the new axis X_{i-1} (axis X'_{i-1}) is in same direction with the axis X_i. The link coordinate system after transformation is shown in Figure 5 (a).

(2) Translate d_i-distance along the axis Z_{i-1} to make the new point O_{i-1}(O'_{i-1}) to the intersection of J_i and the common vertical line of J_i and J_{i+1}, that is, the axis X'_{i-1} is collinear with the axis X_i. The coordinate system of the link after transformation is shown in Figure 5 (b).

(3) Translate the a_i-distance along the axis X'_{i-1} (axis X_i) to make the new point O_{i-1}(O'_{i-1}) move to O_i, coinciding with O_i, the coordinate system of the link after transformation is shown in Figure 5 (c).

(4) Rotate α_i-angle around the axis X'_{i-1} (axis X_i) makes the new axis Z_{i-1} (axis Z'_{i-1}) coincide with the axis Z_i. The link coordinate system after transformation is shown in Figure 5 (d).

So far, the L_{i-1} link coordinate system $O_{i-1}X_{i-1}Y_{i-1}Z_{i-1}$ and the link L_i coordinate system $O_iX_iY_iZ_i$ will completely coincide. The transformation relation between two link coordinate systems can be described by four homogeneous transformation matrices, as shown in Eqs.(1) above.

Where $\cos\theta_i$ will be expressed as $c\theta_i$, and $\sin\theta_i$ will be expressed as $s\theta_i$ for convenience. This agreement will be used in all cases.

3.2.2. Modified D-H Parameter Modeling Method

In the same way, for the link coordinate system established with Modified D-H model above, the link coordinate system L_{i-1} coincides with the link coordinate system L_i after two rotations and two translations, as shown in Figure 4 and Figure 6.

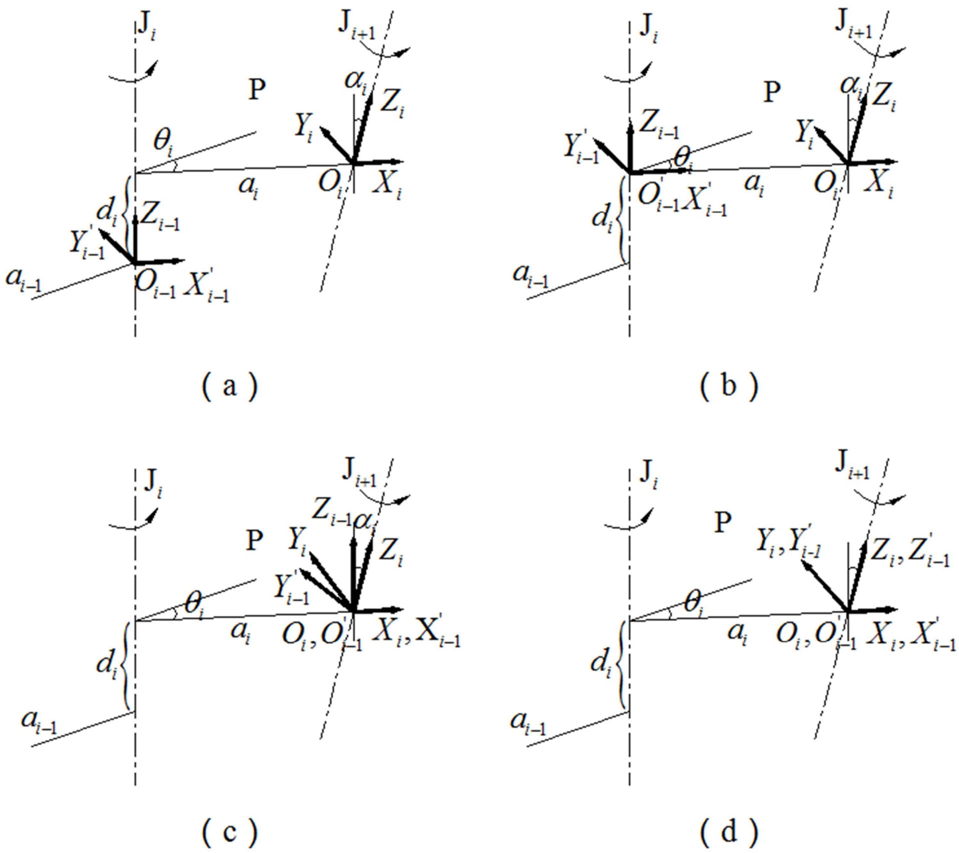

Figure 5. Schematic diagram of link coordinate system transformation (Standard D-H model).

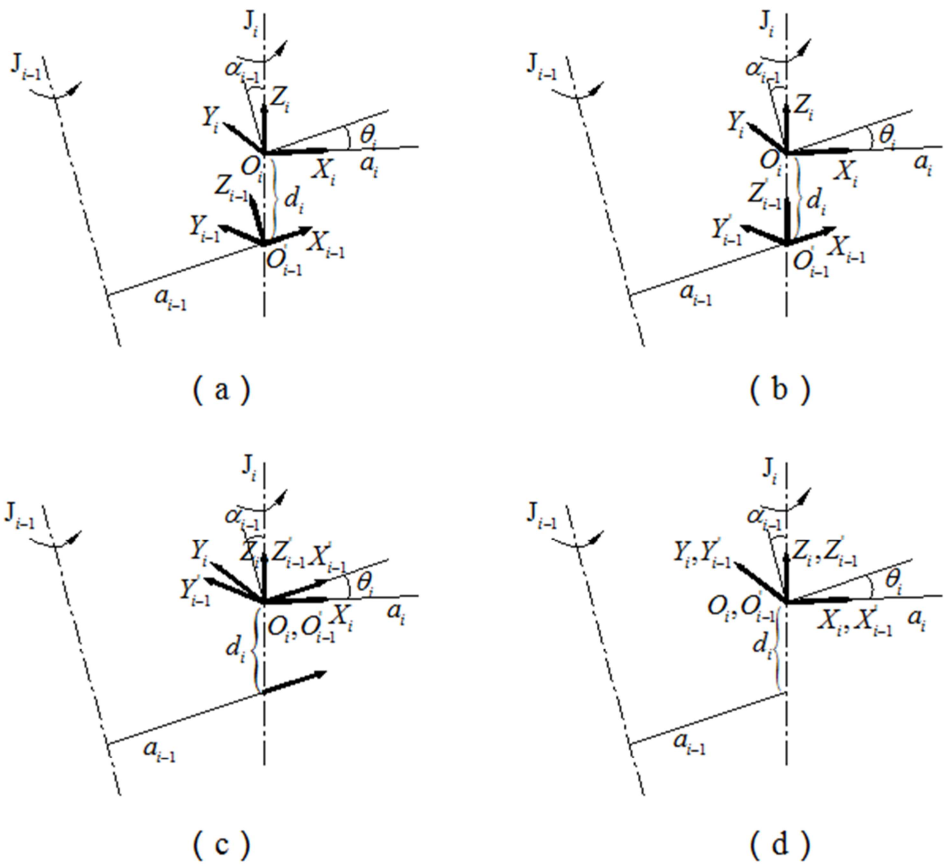

Figure 6. Schematic diagram of link coordinate system transformation (Modified D-H model).

$$
\begin{aligned}
{}^{i\text{-}1}\mathbf{A}_i &= \mathrm{Rot}(Z,\theta_i)\mathrm{Trans}(0,0,d_i)\mathrm{Trans}(a_i,0,0)\mathrm{Rot}(X,\alpha_i) \\
&= \begin{bmatrix} c\theta_i & -s\theta_i & 0 & 0 \\ s\theta_i & c\theta_i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}
\begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c\alpha_i & -s\alpha_i & 0 \\ 0 & s\alpha_i & c\alpha_i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} c\theta_i & -c\alpha_i s\theta_i & s\alpha_i s\theta_i & a_i c\theta_i \\ s\theta_i & c\alpha_i c\theta_i & -s\alpha_i c\theta_i & a_i s\theta_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}
\end{aligned} \tag{1}
$$

$$
\begin{aligned}
{}^{i\text{-}1}\mathbf{A}_i &= \mathrm{Trans}(a_{i-1},0,0)\mathrm{Rot}(X,\alpha_{i-1})\mathrm{Trans}(0,0,d_i)\mathrm{Rot}(Z,\theta_i) \\
&= \begin{bmatrix} 1 & 0 & 0 & a_{i-1} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c\alpha_{i-1} & -s\alpha_{i-1} & 0 \\ 0 & s\alpha_{i-1} & c\alpha_{i-1} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}
\begin{bmatrix} c\theta_i & -s\theta_i & 0 & 0 \\ s\theta_i & c\theta_i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ c\alpha_{i-1}s\theta_i & c\alpha_{i-1}c\theta_i & -s\alpha_{i-1} & -d_i s\alpha_{i-1} \\ s\alpha_{i-1}s\theta_i & s\alpha_{i-1}c\theta_i & c\alpha_{i-1} & d_i c\alpha_{i-1} \\ 0 & 0 & 0 & 1 \end{bmatrix}
\end{aligned} \tag{2}
$$

(1) Translate the a_{i-1}-distance along the axis X_{i-1} to make the point O_{i-1} move to O'_{i-1}, the coordinate system of the link after transformation is shown in Figure 6 (a).

(2) Rotate α_{i-1}-angle around the axis X_{i-1} to make the new axis Z_{i-1} (axis Z'_{i-1}) coincide with the axis Z_i. The coordinate system of the link after transformation is shown in Figure 6 (b).

(3) Translate d_i-distance along the axis Z'_{i-1} (axis Z_i) to make the new point O_{i-1}(O'_{i-1}) to O_i, that is, the axis Z'_{i-1} coincidences with the axis Z_i. The coordinate system of the link after transformation is shown in Figure 6 (c).

(4) Rotate θ_i-degree around the axis Z'_{i-1} (axis Z_i), so that the new axis X_{i-1} (axis X'_{i-1}) is in same direction with the axis X_i. The coordinate system of the link after transformation is shown in Figure 6 (d).

So far, the L_{i-1} link coordinate system $O_{i-1}X_{i-1}Y_{i-1}Z_{i-1}$ and the link L_i coordinate system $O_iX_iY_iZ_i$ will completely coincide. The transformation relation between two link coordinate systems can be described by four homogeneous transformation matrices, as shown in Eqs.(2) below.

4. The Difference of the Two Models

4.1. Transform Order Affects the Results

Take Standard D-H model as an example. In the original rules, the coordinate system $O_{i-1}X_{i-1}Y_{i-1}Z_{i-1}$ is configured at axis J_i, and transforming is along or around axis Z_i, and we get a reasonable result. But if the links transform along or around axis X_i firstly, we will get an unreasonable result shown in the Figure 7. Now the direction of Z_i axis is not parallel to that of d_i parameter, the next transformation is not possible. So we must firstly transform along or around the Z_i axis to make connection of the two adjacent coordinate system originals be parallel with next transform direction a_i. Differently by Modified D-H model rules, the coordinate system $O_iX_iY_iZ_i$ is configured at axis J_i, then transform must be along or around X_i axis firstly[10].

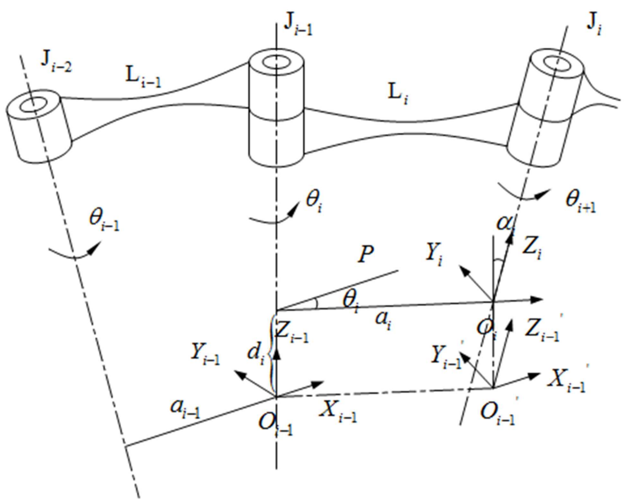

Figure 7. *Transform around X-axis (Standard D-H model).*

4.2. The Order of Rotation Matrix Affects the Result

According to the transform order, homogeneous transformation coordinate system between two link coordinate systems is obtained by multiplying four transformation matrices. It can be seen that the order of four transformation matrices in the Eqs.(1) and Eqs.(2) is different and the subscripts are different, which causes the same result in fact [15]. So how the order of matrix multiplication affects the results?

(1) Multiplication of two rotation matrices. Set a matrix as shown in Eqs.(3) which means the coordinate system rotate θ -angle around axis X , and a matrix as shown in Eqs.(4) which means the coordinate system rotate θ-angle around axis Z . Then calculate the multiplication of two matrices, the Eqs.(5)-(6) are the different results we can see.

$$R(x,\theta)=\begin{bmatrix}1&0&0&0\\0&c\theta&-s\theta&0\\0&s\theta&c\theta&0\\0&0&0&1\end{bmatrix}\tag{3}$$

$$R(z,\theta)=\begin{bmatrix}c\theta&-s\theta&0&0\\s\theta&c\theta&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}\tag{4}$$

$$R(x,\theta)R(z,\theta)=\begin{bmatrix}c\theta&-s\theta&0&0\\c\theta s\theta&c^2\theta&-s\theta&0\\s^2\theta&c\theta s\theta&c\theta&0\\0&0&0&1\end{bmatrix}\tag{5}$$

$$R(z,\theta)R(x,\theta)=\begin{bmatrix}c\theta&-c\theta s\theta&s^2\theta&0\\c\theta s\theta&c^2\theta&-c\theta s\theta&0\\0&s\theta&c\theta&0\\0&0&0&1\end{bmatrix}\tag{6}$$

$$T(x,d)=\begin{bmatrix}1&0&0&d\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}\tag{7}$$

$$T(x,d)R(x,\theta)=\begin{bmatrix}1&0&0&d\\0&c\theta&-s\theta&0\\0&s\theta&c\theta&0\\0&0&0&1\end{bmatrix}\tag{8}$$

$$R(x,\theta)T(x,d)=\begin{bmatrix}1&0&0&d\\0&c\theta&-s\theta&0\\0&s\theta&c\theta&0\\0&0&0&1\end{bmatrix}\tag{9}$$

(2) Multiplication of translation matrix and rotation matrix. Set a matrix as shown in Eqs.(7) which means the coordinate system translate d - displacement along axis X . Then calculate the multiplication of the matrix and the rotation matrix shown in Eqs.(3), and the Eqs.(8)-(9) are the same results we can see.

(3) Multiplication of two translation matrices. Set a matrix as shown in Eqs.(10) which means the coordinate system translate d - displacement along axis Z . Then calculate the multiplication of the matrix and the translation matrix shown in Eqs.(7), and the Eqs.(11)-(12) are the same results we can see.

$$T(z,d)=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&d\\0&0&0&1\end{bmatrix}\tag{10}$$

$$T(x,d)T(z,d)=\begin{bmatrix}1&0&0&d\\0&1&0&0\\0&0&1&d\\0&0&0&1\end{bmatrix}\tag{11}$$

$$T(z,d)T(x,d)=\begin{bmatrix}1&0&0&d\\0&1&0&0\\0&0&1&d\\0&0&0&1\end{bmatrix}\tag{12}$$

In summary, the order of rotation matrix affects the result. But the order of translation matrix affects the result. So when we establish a Standard D-H model, the Eqs.(1) should be chosen but not Eqs.(2).

5. Tips for Quickly Distinguishing Between Two Models

1. Observe the direction of the Z_i axis of the link coordinate system and J_i axis, especially the Z_1 axis. Because the base coordinate system position is uncertain, but the first link coordinate system is certain. The differences of Standard D-H model and Modified D-H model is that Z_1 axis is collinear with J_{i-1} axis or J_i axis. And the first joint of a standard six-degree-of-freedom manipulator is always vertical to the ground, the second joint is vertical to the first joint. So when Z_1 axis is vertical, it is Standard D-H model, or when Z_1 axis is horizontal, it is Modified D-H model.

2. Observe the position of the origin O_i of coordinates, it is arranged on the J_i axis or the J_{i+1} axis. The position of the origin determines whether the rotation order is first along or around the X_i axis or the Z_1 axis. When the subscript of the origin O_i and the J_i axis are different, it is Standard D-H model, or it is Modified D-H model.

3. Observe the transformation matrix of the link coordinate system, especially the subscripts of α and a in the matrix. When there are different subscripts in the transformation matrix, it is Standard D-H model, otherwise it is Modified D-H model.

To sum up, although the difference between Standard D-H model and Modified D-H model seems very slight, the reasons for this difference are essential. Standard D-H model parameters can not be substituted into Modified D-H model transformation matrix, and vice versa. In other words, the link coordinate system established by different models must use corresponding link transformation matrix.

6. Simulation Verification

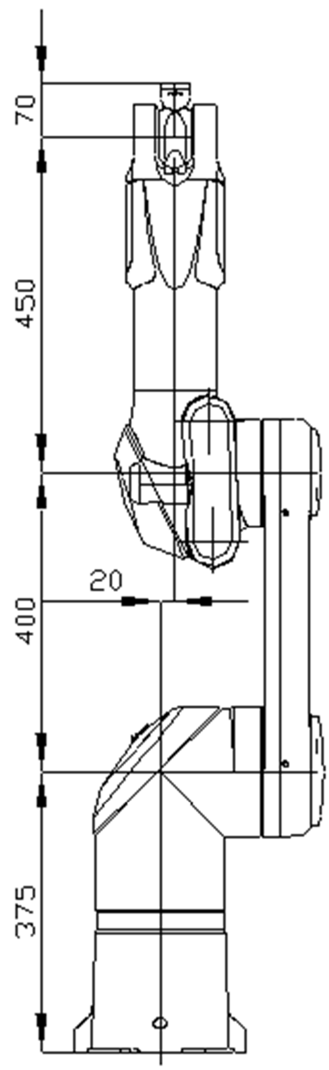

Figure 8. *Staubli TX60L structural parameters.*

Although Standard D-H model and Modified D-H model is different, the results of one same robot are same. That is, the homogeneous transformation matrix of the flange coordinate system at the end of the robot is unique relative to the basic coordinate system of the robot, which will be proved below by the simulation of the robot toolbox in MATLAB.

Table 1. *Staubli TX60L Standard D-H model kinematic parameters.*

link i	$\theta_i(°)$	$d_i(mm)$	$a_i(mm)$	$\alpha_i(°)$
1	$\theta_1(0)$	0	0	-90
2	$\theta_2(-90)$	0	400	0
3	$\theta_3(90)$	20	0	90
4	$\theta_4(0)$	450	0	-90
5	$\theta_5(0)$	0	0	90
6	$\theta_6(0)$	70	0	0

Table 2. *Staubli TX60L Modified D-H model kinematic parameters.*

link i	$\theta_i(°)$	$d_i(mm)$	$a_i(mm)$	$\alpha_i(°)$
1	$\theta_1(0)$	0	0	0
2	$\theta_2(-90)$	20	0	-90
3	$\theta_3(90)$	0	400	0
4	$\theta_4(0)$	450	0	90
5	$\theta_5(0)$	0	0	-90
6	$\theta_6(0)$	70	0	90

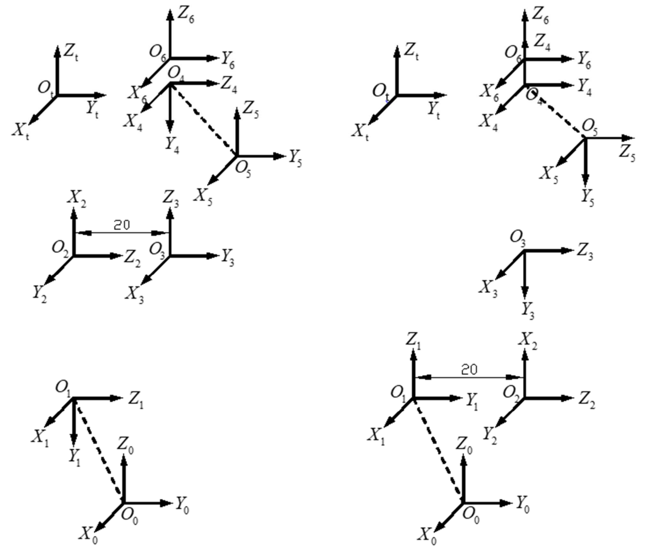

Figure 9. *Standard D-H model (left) and Modified D-H model (right) of Staubli TX60L.*

The object of research in this paper is Swiss Staubli TX60L standard 6-DOF industrial robot whose structural parameters are shown in Figure 8. As described in Section 2, the coordinate systems of links are established by Standard D-H model and Modified D-H model, respectively, as shown in Figure 9. In the figure, the dotted line indicate that the origins are coincident. The origin O_0 (O_1) is at the intersection of the axis J_1 and the axis J_2, and the origin O_4 (O_5) is at the intersection of the axis J_4 and the axis J_5. Frame $O_tX_tY_tZ_t$ is tool coordinate system, whose default attitude is the same as that of the link L_6 coordinate system, with only the offset of the origin position in the three directions of the axis X_6, Y_6, Z_6.

The D-H parameters of the two models are obtained according to the structural parameters of the robot as shown in Table 1-2.

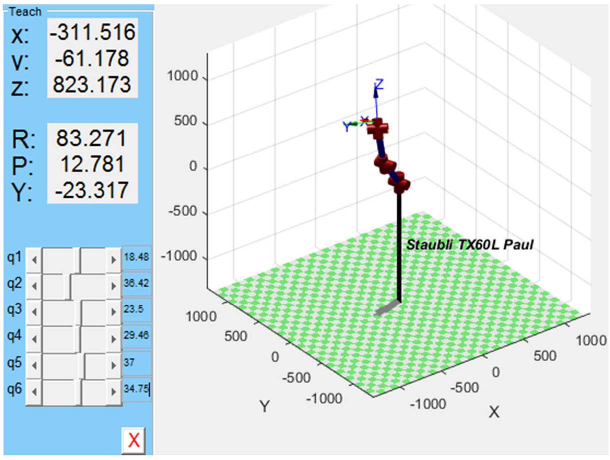

Figure 10. The end pose in Robot Toolbox (Standard D-H model).

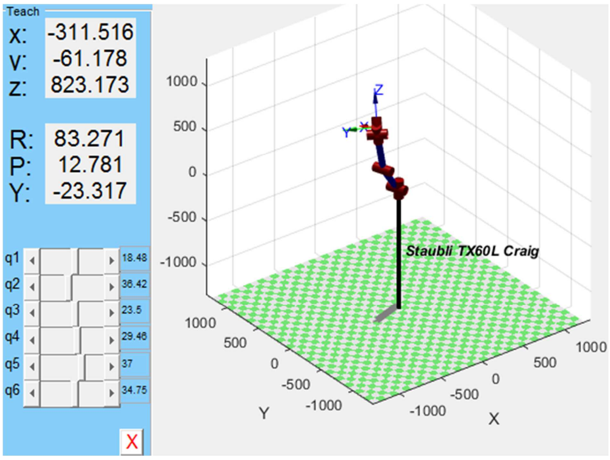

Figure 11. The end pose in Robot Toolbox (Modified D-H model).

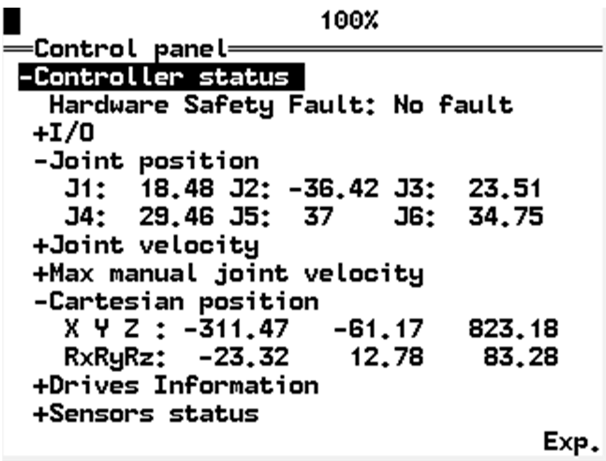

Figure 12. The end pose in Emulator.

Finally, according to the D-H parameters in Tab1-2 respectively, based on MATLAB R2015a, using Robot Toolbox (version: MATLAB_Robotic_Toolbox_v10.1) to write corresponding programs [16, 17], the simulation results are shown as Figures 10-11. At the same time, Emulator, the simulative teaching device in Staubli Robotics Suite (SRS) software, can also control the motion of the robot. And current joint angle value and the position and posture of the end flange in the base coordinate system of the robot can be read through the control panel in the main menu (Figure 12).

Compare the joint angle and position and posture in MATLAB and Emulator. If the results are the same, the simulation model is correct. The results are shown above where we can see the same and correct results.

7. Conclusion

This paper presents and compares the specific steps of Standard D-H modeling method and Modified D-H modeling method in detail. The way of the frames establishing is first compared. The frame which is attached to the *i*th axis by Standard D-H modeling method is *i*-1th frame, or *i*th frame by Modified D-H modeling method. According to the position of the frame and the joint, the order of frame transformation is around or along Z-axis by Standard D-H modeling method, or around or along X-axis by Modified D-H modeling method. Then multiplication of homogeneous transformation matrices in order of frame transformation is final result. The effect of transformation order of homogeneous matrices on final result is studied, that is, the order of translation transformation does not affect the result, but the order of rotation transformation does affect a lot. On the intuitive level, the two methods can be quickly distinguished by observing the subscripts of the link coordinate frame and of the origin of the coordinate frame and of the element in the homogeneous transformation matrix. Finally it should be noted that the most common error in the using process is establishing frame by Standard D-H modeling method and using multiplication of homogeneous transformation matrices as Modified D-H modeling method because of simplicity and uniformity of subscripts. D-H modeling method is the base of researching robot, so it's important to distinguish and use it correctly.

References

[1] Shiming Ji, Xihuan Huang. Overview of the Development and Application of Industrial Robot Technology [J]. Mechanical and Electrical Engineering, 2015, 32 (01): 1-13.

[2] Wang Huiyong, Xingli, Li Xiangman, et al. [J]. Application status and development analysis of industrial robots [J]. Electromechanical technology, 2018, (6): 115-117. DOI: 10.19508/j.cnki. 1672-4801.2018.06.034.

[3] S K SAHA. Introduction to Robotics [M]. Beijing: China Machine Press, 2010: 100-101.

[4] DENAVIT J, HARTENBERG R S. A kinematic notation for lower-pair mechanisms based on matrices [J]. Trans of the Asme. Journal of Applied Mechanics, 1955, 22: 215-221.

[5] WALDRON K J. A study of overconstrained linkage geometry by solution of closure equations — Part 1. Method of study [J]. Mechanism & Machine Theory, 1973, 8 (1): 95-104.

[6] PAUL R P. Robot manipulators: mathematics, programming, and control. The Computer Control of Robot Manipulators [M]. The MIT Press, 1981: 51-53.

[7] CRAIG J J. Introduction to Robotics: Mechanics and Control, 3/E [M]. Pearson Education, Inc, 2005: 64-69.

[8] HE X X, TIAN W, ZENG Y F, et al. Robot positioning error and residual error compensation for aircraff assembly [J]. Acta Aeronautica et Astronautica Sinica, 2017, 38 (4): 420538.

[9] Gaoyi, Ma Guoqing, Yu Zhenglin, Cao Guohua. Kinematics analysis and three-dimensional visualization simulation of a six-degree-of-freedom industrial robot [J]. China Mechanical Engineering, 2016, 27 (13): 1726-1731.

[10] Wang Mulan, Xu Kaiyun, Rao Huaqiu, Zhang Sidi. Inverse kinematics solution of multi-joint robot based on improved BP neural network [J]. Journal of Nanjing University of Aeronautics and Astronautics, 2006 (S1): 83-87.

[11] John J. Craig. Introduction to robotics mechanics and control [M]. Beijing: China Machine Press, 2006: 29-32.

[12] Cai Zixing. Fundamentals of Robotics [M]. Beijing: China Machine Press, 2009.5: 18-22.

[13] Bruno Siciliano, Lorenzo Sciavicco, Luigi Villani, Giuseppe Oriolo. Robotics: modeling, planning and control [M]. Xi'an: Xi'an Jiaotong University Press, 2013.11: 44-48.

[14] Hang Wang, Hanghang Qi, Minghui Xu, et al. Research on the Relationship between Classic Denavit-Hartenberg and Modified Denavit-Hartenberg [C].//2014 Seventh International Symposium on Computational Intelligence and Design, vol. 2: 2014 Seventh International Symposium on Computational Intelligence and Design (ISCID 2014), 13-14 December 2014, Hangzhou, China. 2014: 26-29.n

[15] Ming Tan, De Xu, et al. Advanced Robot Control [M]. Higher Education Press, 2007: 39-46.

[16] (2019) The mathworks wedsite. Available: https://ww2.mathworks.cn/matlabcentral/fileexchange/14886-robotic-toolbox?s_tid=srchtitle

[17] (2019) Petercorke website. Available: http://petercorke.com/wordpress/toolboxes/robotics-toolbox.