Euler - Lagrangian Mechanics on (2,0)-Jet Bundles with Constraints

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Abstract

This paper aims to present Euler -Lagrangian Mechanics formalism for mechanical systems using (2,0)-Jet Bundles with Constraints, which represent an interesting multidisciplinary field of research. As a result of this study, partial differential equations will be obtained for movement of objects in space and solutions of these equations will be made by using the Maple computer program. In this study, some geometrical, relativistic, mechanical, and physical results related to (2,0)-Jet Bundles with Constraints mechanical systems broad applications in mathematical physics, geometrical optics, classical mechanics, analytical mechanics, mechanical systems, thermodynamics, geometric quantization and applied mathematics such as control theory.

Keywords

Geometry of Holomorphic, (2,0) Jet Bundle, Constrained Lagrangian Dynamics

1. Introduction

The study of Lagrangian Dynamics is one of the most important branches of differential Geometry due to its direct correlation with classical physics, and physical sciences and other practical fields, Dynamics systems are the most important topics of differential geometry. It considers regulations the dynamics of Lagrangian topics in modern differential geometry.

In literature, there are a lot of studies about Lagrangian mechanics, formalisms, systems and equations. There are also large number of studies in this subject, for example.

Zalutchi For this study the main topics of this The (2,0)-jet bundles geometry: the fibre structure, its complexified tangent bundle, the decomposition by a nonlinear complex connection, N^jlinear complex connection [1].

W. Stoll developed a general framework for embedded on holomorphic a systematic treatment of the theory of lling by holomorphic Symplectic manifolds [3].

In a set of papers [7-9], are studied the problems of algebraic geometry for aholomorphic jet bundle. The holomorphic jet bundle has a natural structure of complex manifold, whose total space will be further studied by the same schedule as for the real case for the known k- osculator bundle, topic intensively investigated in the last decade, especially by the Romanian geometers.

Audin and Lafontaine introduce to Symplectic geometry and relevant techniques of Riemannian geometry, proofs of theorem, an investigation of local properties of holomorphic curves, including positivity of intersections, and applications to Lagrangian embeddings problems [10].

E. Azizpour. We introduce the concept of a dynamical connection on a graded jet bundle $J^{1,1}(R^{111}(M; A))$ in terms of an almost tangent structure [11].

So, if \mathcal{M} is an m-dimensional configuration manifold and L: $T\mathcal{M} \rightarrow R$ is a regular Lagrangian function, then there is a unique vector field X on $T\mathcal{M}$ such that dynamic equations are determined by

$$i_{X}\phi_{L} = dE_{L} \qquad (1)$$

where ϕ indicates the symplectic form. The triple $(T\mathcal{M}, \phi_L, L)$ is called Lagrangian system on the cotangent bundle $T^*\mathcal{M}$. There are many studies about Lagrangian mechanics, formalisms, systems and equations given by

$$\frac{\mathrm{d}}{\mathrm{dt}} \left(\frac{\partial L}{\partial \dot{q}^{i}} \right) - \frac{\partial L}{\partial q^{i}} = 0 \tag{2}$$

It is said that the quartet $(T\mathcal{M}, \varphi_L, E_L, \sigma)$ defines a mechanical system with constraints if vector field ξ given by the equations of motion

$$i_{\xi_L}\omega_L = dE_L + B^a(\sigma_a)_i = 0 \tag{3}$$

is a second order differential equation. Then, it is given Euler-Lagrange equations with constraints as follows

$$\frac{\mathrm{d}}{\mathrm{dt}} \left(\frac{\partial \mathrm{L}}{\partial \dot{\mathrm{q}}^{\mathrm{i}}} \right) - \frac{\partial \mathrm{L}}{\partial \mathrm{q}^{\mathrm{i}}} = \mathrm{B}^{\mathrm{a}} (\sigma_{\mathrm{a}})_{\mathrm{i}} \tag{4}$$

The present paper is structured as follows introduction In first section preliminaries. In second section, we introduce. In three section Euler - Lagrangian Mechanics with constraints and conclusion, we discuss some geometric-physical results about Lagrangian equations on (2,0)-jet bundles with constraints.

2. The Geometry of $J^{(2,0)}\mathcal{M}$ Bundles

Definition 2.1 [1]

Let \mathcal{M} be a complex manifold, $T_c \mathcal{M} = \mathring{T} \mathcal{M} \oplus \mathring{T} \mathcal{M}$, the complexified tangent bundle of (1,0) - and of (0,1) -type vectors, respectively. If $(z^i)_{i=\overline{1;n}}$ are complex coordinates, then $\mathring{T}_z \mathcal{M}$ is spanned by $\left\{\frac{\partial}{\partial z^i}\right\}_{i=\overline{1;n}}$ and $\mathring{T}_z \mathcal{M}$ is spanned by $\left\{\frac{\partial}{\partial z^i}\right\}_{i=\overline{1;n}}$ moreover $\mathring{T} \mathcal{M}$ is a holomorphic vector bundle.

Let $Z = (z^i, X^i = \eta^{i^{(1)}} = \frac{dz^i}{d\theta}, Y^i = \eta^{i^{(2)}} = \frac{d^2z^i}{d\theta^2})$ be local complex coordinates in the chart $(U; \psi)$ from $J^{(2,0)}\mathcal{M}$; we shall the following notations

$$Z = \left(z^{i}, x^{i} = \eta^{i^{(1)}}, y^{i} = \eta^{i^{(2)}}\right) = (z^{i}, X^{i}, Y^{i})$$
(5)

Theorem 2.2 [1]

A

A local basis in
$$\hat{T}_{z}(J^{(2,0)}\mathcal{M})$$
 is $\left\{\frac{\partial}{\partial z^{i}}, \frac{\partial}{\partial x^{i^{i}}}, \frac{\partial}{\partial y^{i}}\right\}_{i=\overline{1;n}}$ and in

$$\begin{split} &\mathring{T}_{z}\big(J^{(2,0)}\mathcal{M}\big)\text{theirs}\\ &\text{conjugates}\,\Big\{\frac{\partial}{\partial\bar{z^{i}}},\frac{\partial}{\partial\bar{x^{i}}},\frac{\partial}{\partial\bar{y^{i}}}\Big\}_{i=\overline{1;n}}: \text{ Due to holomorphic changes}\\ &\text{on }J^{(2,0)}\mathcal{M} \text{ , that is all of }\frac{\partial \dot{z^{i}}}{\partial \overline{z^{j}}},\frac{\partial \dot{x^{i}}}{\partial \overline{z^{j}}},\frac{\partial \dot{y^{i}}}{\partial \overline{x^{j}}},\frac{\partial \dot{y^{i}}}{\partial \overline{x^{j}}},\frac{\partial \dot{y^{i}}}{\partial \overline{y^{j}}},\frac{\partial \dot{$$

$$\frac{\partial}{\partial z^{j}} = \frac{\partial \dot{z^{i}}}{\partial z^{j}} \frac{\partial}{\partial \dot{z^{i}}} + \frac{\partial \dot{x^{i}}}{\partial z^{j}} \frac{\partial}{\partial \dot{x^{i}}} + \frac{\partial \dot{y^{i}}}{\partial z^{j}} \xrightarrow{\partial} \\ \frac{\partial}{\partial x^{j}} = \frac{\partial \dot{x^{i}}}{\partial z^{j}} \frac{\partial}{\partial \dot{x^{i}}} + \frac{\partial \dot{y^{i}}}{\partial z^{j}} \frac{\partial}{\partial \dot{y^{i}}} \\ \frac{\partial}{\partial x^{j}} = \frac{\partial \dot{y^{i}}}{\partial z^{j}} \xrightarrow{\partial} \\ \frac{\partial}{\partial x^{j}} = \frac{\partial \dot{y^{i}}}{\partial z^{j}} \xrightarrow{\partial} \\ \frac{\partial}{\partial z^{j}} = \frac{\partial \dot{z}^{j}}{\partial z^{j}} \xrightarrow{\partial} \\ \frac{\partial}{\partial z^{j}} = \frac{\partial}{\partial z^{j}} = \frac{\partial}{\partial z^{j}} \xrightarrow{\partial} \\ \frac{\partial}{\partial z^{j}} = \frac{\partial}{\partial z^{j}} \xrightarrow{\partial} \\ \frac{\partial}{\partial z^{j}} = \frac{\partial}{\partial z^{j}} \xrightarrow{\partial} \\ \frac{\partial}{\partial z^{j}} = \frac{\partial}{\partial z^{j}$$

Infer that $\frac{\partial \dot{z}^{i}}{\partial z^{j}} = \frac{\partial \dot{x}^{i}}{\partial x^{j}} = \frac{\partial \dot{y}^{i}}{\partial y^{j}}$ but in change $\frac{\partial \dot{z}^{i}}{\partial z^{j}} = \frac{\partial \dot{x}^{i}}{\partial x^{j}}$ contain the second order derivatives of \dot{z}^{i} . While $\frac{\partial \dot{x}^{i}}{\partial z^{j}}$ contains even the

3-th derivatives of \dot{z}^i .

Theorem 2.3

On $T_c(J^{(2,0)}\mathcal{M})$ the natural complex structure $J^2 = -I$ acts as follows:

$$J\left(\frac{\partial}{\partial z^{j}}\right) = i\frac{\partial}{\partial z^{j}}, J\left(\frac{\partial}{\partial x^{j}}\right) = i\frac{\partial}{\partial x^{j}}, J\left(\frac{\partial}{\partial y^{j}}\right) = i\frac{\partial}{\partial y^{j}}$$
$$J\left(\frac{\partial}{\partial \bar{z}^{j}}\right) = -i\frac{\partial}{\partial z^{j}}, J\left(\frac{\partial}{\partial \bar{x}^{j}}\right) = -i\frac{\partial}{\partial x^{j}}, J\left(\frac{\partial}{\partial \bar{y}^{j}}\right) = -i\frac{\partial}{\partial y^{j}}$$
(7)

Lemma 2.4[2] If ω , θ and k –formbe respectively then:-

$$d\omega \wedge d\theta = -d\theta \wedge d\omega$$
$$d^{2} = d \circ d = 0$$
$$d(\omega \wedge \psi) = d\omega \wedge \psi + (-1)^{k} d\psi \wedge \omega$$

Lemma 2.5 [2] Suppose $(U, x_1, ..., x_n)$ is a chart on a manifold. Then

$$\left(\frac{\partial x^{j}}{\partial x^{i}}\right) = \delta^{i}_{j} = \begin{cases} 1, i = j\\ 0, i \neq j \end{cases}$$
(8)

Definition 2.6An exterior differentiation or exterior derivative on a manifold \mathcal{M} is an R-linear map

d:
$$\Omega^*(\mathcal{M}) \to \Omega^*(\mathcal{M})$$

Then the exterior derivative of ω is (k + 1) – form given by

$$d\omega = \sum_{i=1}^{m} d(\omega_{i_1,\dots,i_k}) dx^{i_1} \wedge dx^{i_2} \wedge \dots \wedge dx^{i_k} Wherek > 0 \ (9)$$

Definition 2. 7 [3] A Symplectic structure on an even dimensional manifold \mathcal{M} is a 2-form ω on \mathcal{M} satisfying:-

(i) $d\omega = 0$, i.e., ω is a closed form.

(ii) wis non degenerate

3. Euler - Lagrangian Mechanics Equations

In this section, we obtain complex Euler -Lagrangian equations for classical mechanics structured on momentum space $T_c(J^{(2,0)}\mathcal{M})$ that is 2m- dimensional tangent bundle of an m-dimensional configuration manifold \mathcal{M} .

Definition 3.1

Let map L: $T\mathcal{M} \to \mathcal{M}$ such that

$$= T - P \tag{10}$$

The Lagrangian function, where we find that

L

T= Kinetic energy,

P = Potential energy

where φ_L is the symplectic form and E_L is the energy associated to L

Let *J* be an almost complex structure on the $T_c \mathcal{M}$ and $(z^i, \bar{z}^i, x^i, \bar{x}^i, y^i, \bar{y}^i)$ its complex coordinates. Assume to be semispray to the vector field ξ given as:

$$\xi = \xi_{\rm L} = \xi^{i} \frac{\partial}{\partial z^{i}} + \overline{\xi}^{i} \frac{\partial}{\partial \overline{z}^{i}} + \eta^{i} \frac{\partial}{\partial x^{i}} + \overline{\eta}^{i} \frac{\partial}{\partial \overline{x}^{i}} + \zeta^{i} \frac{\partial}{\partial y^{i}} + \overline{\zeta}^{i} \frac{\partial}{\partial \overline{y}^{i}} (11)$$

$$\xi^{i} = \dot{z}^{i} = \bar{z}^{i}, \bar{\xi}^{i} = \dot{\xi}^{i} = \dot{z}^{i} = \dot{z}^{i}$$
 $\zeta^{i} = \dot{y}^{i} = \bar{y}^{i}, \bar{\zeta}^{i} = \dot{\zeta}^{i}$

$$\eta^{i} = \dot{x}^{i} = \bar{x}^{i}, \bar{\eta}^{i} = \dot{\eta}^{i} = \ddot{x}^{i} = \dot{\bar{x}}^{i}$$

The vector field determined by

$$V = J\xi_L = J\left(\xi^i \frac{\partial}{\partial z^i} + \bar{\xi}^i \frac{\partial}{\partial \bar{z}^i} + \eta^i \frac{\partial}{\partial x^i} + \bar{\eta}^i \frac{\partial}{\partial \bar{x}^i} + \zeta^i \frac{\partial}{\partial y^i} + \bar{\zeta}^i \frac{\partial}{\partial \bar{y}^i}\right) (12)$$

 $= \ddot{y}^i = \overline{\dot{y}}^i$

is called Liouville vector field on the complex manifold $J^{(2,0)}\mathcal{M}$. The closed 2-form given by $\phi_L = -dd_J L$ such that

$$d_{J} = i\frac{\partial}{\partial z^{i}} - i\frac{\partial}{\partial \overline{z}^{i}} + i\frac{\partial}{\partial x^{i}} - i\frac{\partial}{\partial \overline{x}^{i}} + i\frac{\partial}{\partial y^{i}} - i\frac{\partial}{\partial \overline{y}^{i}} : \mathcal{F}(J^{(2,0)}\mathcal{M}) \longrightarrow \Lambda^{1}T(J^{(2,0)}\mathcal{M})$$

$$\phi_{L} = -dd_{J}L = -d\left(i\frac{\partial L}{\partial z^{i}} - i\frac{\partial L}{\partial \overline{z}^{i}} + i\frac{\partial L}{\partial x^{i}} - i\frac{\partial L}{\partial \overline{x}^{i}} + i\frac{\partial L}{\partial y^{i}} - i\frac{\partial L}{\partial \overline{y}^{i}}\right)$$
(13)

By means of (1) Euler -Lagrangian Mechanics on (2,0)-jet bundles is found the following as

$$\frac{\partial L}{\partial z^{i}} - i \frac{\partial}{\partial t} \left(\frac{\partial L}{\partial z^{i}} \right) = 0, \frac{\partial L}{\partial \dot{z}^{i}} + i \frac{\partial}{\partial t} \left(\frac{\partial L}{\partial \dot{z}^{i}} \right) = 0$$
$$\frac{\partial L}{\partial x^{i}} - i \frac{\partial}{\partial t} \left(\frac{\partial L}{\partial x^{i}} \right) = 0, \frac{\partial L}{\partial \dot{x}^{i}} + i \frac{\partial}{\partial t} \left(\frac{\partial L}{\partial \dot{x}^{i}} \right) = 0$$
$$\frac{\partial L}{\partial y^{i}} - i \frac{\partial}{\partial t} \left(\frac{\partial L}{\partial y^{i}} \right) = 0, \frac{\partial L}{\partial \dot{y}^{i}} + i \frac{\partial}{\partial t} \left(\frac{\partial L}{\partial \dot{y}^{i}} \right) = 0$$
(14)

4. Euler - Lagrangian Mechanics Equations with Constraints

In this section, we obtain complex Euler -Lagrangian equations with constraints for classical mechanics structured on momentum space $T_c(J^{(2,0)}\mathcal{M})$ that is 2m- dimensional tangent bundle of an m-dimensional configuration manifold \mathcal{M} .

Let *J* be an almost complex structure on the $T_c \mathcal{M}$ and $(z^i, \bar{z}^i, x^i, \bar{x}^i, y^i, \bar{y}^i)$ its complex coordinates. Assume to be semispray to the vector field ξ given as:

$$\xi = \xi_{\rm L} = \xi^{\rm i} \frac{\partial}{\partial z^{\rm i}} + \bar{\xi}^{\rm i} \frac{\partial}{\partial \bar{z}^{\rm i}} + \eta^{\rm i} \frac{\partial}{\partial x^{\rm i}} + \bar{\eta}^{\rm i} \frac{\partial}{\partial \bar{x}^{\rm i}} + \zeta^{\rm i} \frac{\partial}{\partial y^{\rm i}} + \bar{\zeta}^{\rm i} \frac{\partial}{\partial \bar{y}^{\rm i}} + B^{a} \sigma_{a}, 1 \le a \le n$$

$$\xi^{\rm i} = \dot{z}^{i} = \bar{z}^{i}, \bar{\xi}^{\rm i} = \dot{\xi}^{\rm i} = \ddot{z}^{i} = \dot{z}^{i}$$

$$\eta^{\rm i} = \dot{x}^{i} = \bar{x}^{i}, \bar{\eta}^{\rm i} = \dot{\eta}^{\rm i} = \ddot{x}^{i} = \dot{z}^{i}$$

$$\zeta^{\rm i} = \dot{y}^{i} = \bar{y}^{i}, \bar{\zeta}^{\rm i} = \dot{\zeta}^{\rm i} = \dot{y}^{i} = \dot{y}^{i}$$

$$(15)$$

The vector field determined by

$$V = J\xi_{L} = J\left(\xi^{i}\frac{\partial}{\partial z^{i}} + \bar{\xi}^{i}\frac{\partial}{\partial \bar{z}^{i}} + \eta^{i}\frac{\partial}{\partial x^{i}} + \bar{\eta}^{i}\frac{\partial}{\partial \bar{x}^{i}} + \zeta^{i}\frac{\partial}{\partial y^{i}} + \bar{\zeta}^{i}\frac{\partial}{\partial \bar{y}^{i}}\right)$$

$$V = J\xi_{L} = J\xi^{i}\frac{\partial}{\partial z^{i}} + J\bar{\xi}^{i}\frac{\partial}{\partial \bar{z}^{i}} + J\eta^{i}\frac{\partial}{\partial x^{i}} + J\bar{\eta}^{i}\frac{\partial}{\partial \bar{x}^{i}} + J\bar{\zeta}^{i}\frac{\partial}{\partial y^{i}} + J\bar{\zeta}^{i}\frac{\partial}{\partial \bar{y}^{i}}$$

$$V = J\xi_{L} = \xi^{i}J\left(\frac{\partial}{\partial z^{i}}\right) + \bar{\xi}^{i}J\left(\frac{\partial}{\partial \bar{z}^{i}}\right) + \eta^{i}J\left(\frac{\partial}{\partial x^{i}}\right) + \bar{\eta}^{i}J\left(\frac{\partial}{\partial \bar{x}^{i}}\right) + \zeta^{i}J\left(\frac{\partial}{\partial y^{i}}\right) + \bar{\zeta}^{i}J\left(\frac{\partial}{\partial \bar{y}^{i}}\right)$$

$$V = J\xi_{L} = i\xi^{i}\frac{\partial}{\partial z^{i}} - i\bar{\xi}^{i}\frac{\partial}{\partial \bar{z}^{i}} + i\eta^{i}\frac{\partial}{\partial x^{i}} - i\bar{\eta}^{i}\frac{\partial}{\partial \bar{x}^{i}} + i\zeta^{i}\frac{\partial}{\partial y^{i}} - i\bar{\zeta}^{i}\frac{\partial}{\partial \bar{y}^{i}}$$

$$(16)$$

is called Liouville vector field on the complex manifold $J^{(2,0)}\mathcal{M}$. The closed 2-form given by $\phi_L = -dd_J L$ such that

$$d_{J} = i\frac{\partial}{\partial z^{i}} - i\frac{\partial}{\partial \bar{z}^{i}} + i\frac{\partial}{\partial x^{i}} - i\frac{\partial}{\partial \bar{x}^{i}} + i\frac{\partial}{\partial y^{i}} - i\frac{\partial}{\partial \bar{y}^{i}}: \mathcal{F}(J^{(2,0)}\mathcal{M}) \longrightarrow \Lambda^{1}T(J^{(2,0)}\mathcal{M})$$
(17)

Or

$$d_J L = \left(i\frac{\partial}{\partial z^i} - i\frac{\partial}{\partial \overline{z}^i} + i\frac{\partial}{\partial x^i} - i\frac{\partial}{\partial \overline{x}^i} + i\frac{\partial}{\partial y^i} - i\frac{\partial}{\partial \overline{y}^i}\right)L$$

$$d_{J}L = i\frac{\partial L}{\partial z^{i}} - i\frac{\partial L}{\partial \overline{z}^{i}} + i\frac{\partial L}{\partial x^{i}} - i\frac{\partial L}{\partial \overline{x}^{i}} + i\frac{\partial L}{\partial y^{i}} - i\frac{\partial L}{\partial \overline{y}^{i}}$$
(18)
$$-dd_{J}L = -d\left(i\frac{\partial L}{\partial z^{i}} - i\frac{\partial L}{\partial \overline{z}^{i}} + i\frac{\partial L}{\partial \overline{x}^{i}} - i\frac{\partial L}{\partial \overline{x}^{i}} + i\frac{\partial L}{\partial y^{i}} - i\frac{\partial L}{\partial \overline{y}^{i}}\right)$$

$$\phi_{L} = -dd_{J}L = -d\left(i\frac{\partial L}{\partial \overline{z}^{i}} - i\frac{\partial L}{\partial \overline{z}^{i}} + i\frac{\partial L}{\partial \overline{x}^{i}} - i\frac{\partial L}{\partial \overline{x}^{i}} + i\frac{\partial L}{\partial \overline{y}^{i}} - i\frac{\partial L}{\partial \overline{y}^{i}}\right)$$

$$\phi_{L} = -d\left(i\frac{\partial L}{\partial \overline{z}^{i}} - i\frac{\partial L}{\partial \overline{z}^{i}}\right) - d\left(i\frac{\partial L}{\partial x^{i}} - i\frac{\partial L}{\partial \overline{x}^{i}}\right) - d\left(i\frac{\partial L}{\partial y^{i}} - i\frac{\partial L}{\partial \overline{y}^{i}}\right)$$

is found to be

$$\begin{split} \phi_{L} &= i \frac{\partial^{2} L}{\partial z^{j} \partial z^{i}} dz^{i} \wedge dz^{j} + i \frac{\partial^{2} L}{\partial \overline{z}^{j} \partial z^{i}} dz^{i} \wedge d\overline{z}^{j} + i \frac{\partial^{2} L}{\partial z^{j} \partial \overline{z}^{i}} dz^{j} \wedge d\overline{z}^{i} + i \frac{\partial^{2} L}{\partial \overline{z}^{j} \partial \overline{z}^{i}} d\overline{z}^{j} \wedge d\overline{z}^{i} \\ &+ i \frac{\partial^{2} L}{\partial x^{j} \partial x^{i}} dx^{i} \wedge dx^{j} + i \frac{\partial^{2} L}{\partial \overline{x}^{j} \partial x^{i}} dx^{i} \wedge d\overline{x}^{j} + i \frac{\partial^{2} L}{\partial x^{j} \partial \overline{x}^{i}} dx^{j} \wedge d\overline{x}^{i} + i \frac{\partial^{2} L}{\partial \overline{x}^{j} \partial \overline{x}^{i}} d\overline{x}^{j} \wedge d\overline{x}^{i} \\ &+ i \frac{\partial^{2} L}{\partial y^{j} \partial y^{i}} dy^{i} \wedge dy^{j} + i \frac{\partial^{2} L}{\partial \overline{y}^{j} \partial y^{i}} dy^{j} \wedge d\overline{y}^{j} + i \frac{\partial^{2} L}{\partial \overline{y}^{j} \partial \overline{y}^{i}} dy^{j} \wedge d\overline{y}^{i} \\ &+ i \frac{\partial^{2} L}{\partial \overline{z}^{j} \partial \overline{z}^{i}} dx^{i} \wedge dz^{j} + i \frac{\partial^{2} L}{\partial \overline{y}^{j} \partial \overline{y}^{i}} dy^{j} \wedge d\overline{y}^{i} + i \frac{\partial^{2} L}{\partial \overline{y}^{j} \partial \overline{y}^{i}} d\overline{y}^{j} \wedge d\overline{y}^{i} \end{split}$$
(19)
$$&= (i \frac{\partial^{2} L}{\partial \overline{z}^{j} \partial \overline{z}^{i}} dz^{i} \wedge dz^{j} + i \frac{\partial^{2} L}{\partial \overline{z}^{j} \partial \overline{z}^{i}} dz^{i} \wedge d\overline{z}^{j} + i \frac{\partial^{2} L}{\partial \overline{z}^{j} \partial \overline{z}^{i}} d\overline{z}^{j} \wedge d\overline{z}^{i} \\ &+ i \frac{\partial^{2} L}{\partial \overline{z}^{j} \partial \overline{z}^{i}} dx^{i} \wedge dz^{j} + i \frac{\partial^{2} L}{\partial \overline{z}^{j} \partial \overline{z}^{i}} dx^{i} \wedge d\overline{z}^{j} + i \frac{\partial^{2} L}{\partial \overline{z}^{j} \partial \overline{z}^{i}} dx^{j} \wedge d\overline{z}^{i} \\ &+ i \frac{\partial^{2} L}{\partial x^{j} \partial x^{i}} dx^{i} \wedge dx^{j} + i \frac{\partial^{2} L}{\partial \overline{x}^{j} \partial \overline{x}^{i}} dx^{j} \wedge d\overline{x}^{i} + i \frac{\partial^{2} L}{\partial \overline{x}^{j} \partial \overline{x}^{i}} dx^{j} \wedge d\overline{x}^{i} \\ &+ i \frac{\partial^{2} L}{\partial y^{j} \partial y^{i}} dy^{i} \wedge dy^{j} + i \frac{\partial^{2} L}{\partial \overline{y}^{j} \partial y^{i}} dy^{i} \wedge d\overline{y}^{j} + i \frac{\partial^{2} L}{\partial \overline{x}^{j} \partial \overline{x}^{i}} dx^{j} \wedge d\overline{y}^{i} \\ &+ i \frac{\partial^{2} L}{\partial \overline{y}^{j} \partial \overline{y}^{i}} d\overline{y}^{j} \wedge d\overline{y}^{i} \right) (i\overline{\xi}^{i} \frac{\partial}{\partial \overline{z}^{i}} - i\overline{\xi}^{i} \frac{\partial}{\partial \overline{z}^{i}} + i\eta^{i} \frac{\partial}{\partial \overline{x}^{i}} - i\overline{\eta}^{i} \frac{\partial}{\partial \overline{x}^{i}} + i\overline{\zeta}^{i} \frac{\partial}{\partial \overline{y}^{j}} - i\overline{\zeta}^{i} \frac{\partial}{\partial \overline{y}^{i}} \right)$$

Let ξ be the semispray given by (19) and

$$i_{\bar{\xi}}\phi_{L} = i\xi^{i}\frac{\partial^{2}L}{\partial z^{j}\partial z^{i}}dz^{j} - i\xi^{i}\frac{\partial^{2}L}{\partial z^{j}\partial z^{i}}\delta^{j}_{i}dz^{i} + i\xi^{i}\frac{\partial^{2}L}{\partial \bar{z}^{j}\partial z^{i}}d\bar{z}^{j} - i\bar{\xi}^{i}\frac{\partial^{2}L}{\partial \bar{z}^{j}\partial z^{i}}\delta^{j}_{i}dz^{i}$$

$$+ i\xi^{i}\frac{\partial^{2}L}{\partial z^{j}\partial \bar{z}^{i}}\delta^{j}_{i}d\bar{z}^{i} - i\bar{\xi}^{i}\frac{\partial^{2}L}{\partial z^{j}\partial \bar{z}^{i}}dz^{j} + i\bar{\xi}^{i}\frac{\partial^{2}L}{\partial \bar{z}^{j}\partial \bar{z}^{i}}\delta^{j}_{i}d\bar{z}^{i} - i\bar{\xi}^{i}\frac{\partial^{2}L}{\partial \bar{z}^{j}\partial \bar{z}^{i}}d\bar{z}^{j} + i\eta^{i}\frac{\partial^{2}L}{\partial x^{j}\partial x^{i}}dx^{j}$$

$$- i\eta^{i}\frac{\partial^{2}L}{\partial x^{j}\partial x^{i}}\delta^{j}_{i}dx^{i} + i\eta^{i}\frac{\partial^{2}L}{\partial \bar{x}^{j}\partial x^{i}}d\bar{x}^{j} - i\bar{\eta}^{i}\frac{\partial^{2}L}{\partial \bar{x}^{j}\partial x^{i}}\delta^{j}_{i}dx^{i} + i\eta^{i}\frac{\partial^{2}L}{\partial x^{j}\partial x^{i}}d\bar{x}^{j} - i\bar{\eta}^{i}\frac{\partial^{2}L}{\partial \bar{x}^{j}\partial x^{i}}\delta^{j}_{i}dx^{i} + i\eta^{i}\frac{\partial^{2}L}{\partial x^{j}\partial x^{i}}\delta^{j}_{i}d\bar{x}^{i}$$

$$- i\bar{\eta}^{i}\frac{\partial^{2}L}{\partial x^{j}\partial \bar{x}^{i}}dx^{j} + i\bar{\eta}^{i}\frac{\partial^{2}L}{\partial \bar{x}^{j}\partial \bar{x}^{i}}\delta^{j}_{i}d\bar{x}^{i} - i\bar{\eta}^{i}\frac{\partial^{2}L}{\partial \bar{x}^{j}\partial \bar{x}^{i}}\delta^{j}_{i}d\bar{x}^{j}$$

$$i\zeta^{i}\frac{\partial^{2}L}{\partial x^{j}\partial \bar{x}^{i}}dy^{j} - i\zeta^{i}\frac{\partial^{2}L}{\partial y^{j}\partial y^{i}}\delta^{j}_{i}dy^{i} + i\zeta^{i}\frac{\partial^{2}L}{\partial \bar{y}^{j}\partial y^{i}}d\bar{y}^{j} - i\bar{\zeta}^{i}\frac{\partial^{2}L}{\partial \bar{y}^{j}\partial y^{i}}\delta^{j}_{i}d\bar{y}^{i}$$

$$- i\bar{\zeta}^{i}\frac{\partial^{2}L}{\partial y^{j}\partial y^{i}}dy^{j} + i\bar{\zeta}^{i}\frac{\partial^{2}L}{\partial \bar{y}^{j}\partial y^{i}}\delta^{j}_{i}d\bar{y}^{i} - i\bar{\zeta}^{i}\frac{\partial^{2}L}{\partial \bar{y}^{j}\partial y^{i}}d\bar{y}^{j}$$

$$(20)$$

Since the closed Kahlerian form ϕ_L on $T\mathcal{M}$ is symplectic structure, it is obtained

$$E_{L} = \bar{\xi} - L$$

$$E_{L} = i\bar{\xi}^{i}\frac{\partial}{\partial z^{i}} - i\bar{\xi}^{i}\frac{\partial}{\partial \bar{z}^{i}} + i\eta^{i}\frac{\partial}{\partial x^{i}} - i\bar{\eta}^{i}\frac{\partial}{\partial \bar{x}^{i}} + i\zeta^{i}\frac{\partial}{\partial y^{i}} - i\bar{\zeta}^{i}\frac{\partial}{\partial \bar{y}^{i}} - L$$
(21)

Differential equation (21) we get

$$dE_{L} + B^{a}\sigma_{a} = d\left(i\xi^{i}\frac{\partial}{\partial z^{i}} - i\overline{\xi}^{i}\frac{\partial}{\partial\overline{z}^{i}} + i\eta^{i}\frac{\partial}{\partial x^{i}} - i\overline{\eta}^{i}\frac{\partial}{\partial\overline{x}^{i}} + i\zeta^{i}\frac{\partial}{\partial y^{i}} - i\overline{\zeta}^{i}\frac{\partial}{\partial\overline{y}^{i}} - L\right) + B^{a}(\sigma_{a})_{i}$$

$$dE_{L} + B^{a}\sigma_{a} = i\xi^{i}\frac{\partial^{2}L}{\partial z^{j}\partial z^{i}}dz^{j} - i\overline{\xi}^{i}\frac{\partial^{2}L}{\partial z^{j}\partial\overline{z}^{i}}dz^{j} - \frac{\partial L}{\partial z^{j}\partial\overline{z}^{i}}dz^{j} + i\xi^{i}\frac{\partial^{2}L}{\partial\overline{z}^{j}\partial\overline{z}^{i}}\overline{z}^{j} - i\overline{\xi}^{i}\frac{\partial^{2}L}{\partial z^{j}\partial\overline{z}^{i}}d\overline{z}^{j}$$

$$-\frac{\partial L}{\partial\overline{z}^{j}}d\overline{z}^{j} + i\eta^{i}\frac{\partial^{2}L}{\partial x^{j}\partial x^{i}}dx^{j} - i\overline{\eta}^{i}\frac{\partial^{2}L}{\partial x^{j}\partial\overline{x}^{i}}dx^{j} - \frac{\partial L}{\partial x^{j}\partial\overline{x}^{i}}dx^{j} + i\eta^{i}\frac{\partial^{2}L}{\partial\overline{x}^{j}\partial\overline{x}^{i}}\overline{z}^{j} - i\eta^{i}\frac{\partial^{2}L}{\partial x^{j}\partial\overline{x}^{i}}d\overline{x}^{j} - \frac{\partial L}{\partial\overline{x}^{j}}d\overline{x}^{j}$$

$$+i\zeta^{i}\frac{\partial^{2}L}{\partial y^{j}\partial y^{i}}dy^{j} - i\overline{\zeta}^{i}\frac{\partial^{2}L}{\partial y^{j}\partial\overline{y}^{i}}dy^{j} + i\zeta^{i}\frac{\partial^{2}L}{\partial\overline{y}^{j}\partial\overline{y}^{i}}\overline{y}^{j} - i\overline{\zeta}^{i}\frac{\partial^{2}L}{\partial\overline{y}^{j}}\overline{y}^{j}d\overline{y}^{j} - \frac{\partial L}{\partial\overline{x}^{j}}d\overline{x}^{j}$$

$$(22)$$

With respect to (3), if (21) and (20) is equalized, it is calculated as follows:

$$-i\xi^{i}\frac{\partial^{2}L}{\partial z^{j}\partial z^{i}}dz^{j} - i\overline{\xi}^{i}\frac{\partial^{2}L}{\partial \overline{z}^{j}\partial z^{i}}dz^{j} + \frac{\partial L}{\partial z^{j}}dz^{j} + i\xi^{i}\frac{\partial^{2}L}{\partial z^{j}\partial \overline{z}^{i}}\overline{z}^{j} + i\overline{\xi}^{i}\frac{\partial^{2}L}{\partial \overline{z}^{j}\partial \overline{z}^{i}}d\overline{z}^{j} + \frac{\partial L}{\partial \overline{z}^{j}}d\overline{z}^{j}$$
$$-i\eta^{i}\frac{\partial^{2}L}{\partial x^{j}\partial x^{i}}dx^{j} - i\overline{\eta}^{i}\frac{\partial^{2}L}{\partial \overline{x}^{j}\partial x^{i}}dx^{j} + \frac{\partial L}{\partial x^{j}}dx^{j} + i\eta^{i}\frac{\partial^{2}L}{\partial x^{j}\partial \overline{x}^{i}}\overline{z}^{j} + i\overline{\eta}^{i}\frac{\partial^{2}L}{\partial \overline{x}^{j}\partial \overline{x}^{i}}d\overline{x}^{j} + \frac{\partial L}{\partial \overline{x}^{j}}d\overline{x}^{j}$$
$$-i\overline{\zeta}^{i}\frac{\partial^{2}L}{\partial y^{j}\partial y^{i}}dy^{j} - i\overline{\zeta}^{i}\frac{\partial^{2}L}{\partial \overline{y}^{j}\partial y^{i}}dy^{j} + \frac{\partial L}{\partial y^{j}}dy^{j} + i\overline{\zeta}^{i}\frac{\partial^{2}L}{\partial \overline{y}^{j}\partial \overline{y}^{i}}\overline{y}^{j} + i\overline{\zeta}^{i}\frac{\partial^{2}L}{\partial \overline{y}^{j}\partial \overline{y}^{i}}d\overline{y}^{j} + \frac{\partial L}{\partial \overline{y}^{j}}d\overline{y}^{j}$$
$$= B^{z}\sigma_{a}dz^{j} + B^{z}\dot{\sigma}_{a}d\dot{z}^{j} + B^{x}\sigma_{a}dx^{j} + B^{x}\dot{\sigma}_{a}d\dot{x}^{j} + B^{y}\sigma_{a}dy^{j} + B^{y}\dot{\sigma}_{a}d\dot{y}^{j}$$

Now, let the curve $\alpha: \mathcal{C} \to T\mathcal{M}$ be integral curve of ξ , which satisfies equations

$$-i\left[\xi^{j}\frac{\partial^{2}L}{\partial z^{j}\partial z^{i}} + \dot{\xi}^{i}\frac{\partial^{2}L}{\partial \dot{z}^{j}\partial z^{i}}\right]dz^{j} + \frac{\partial L}{\partial z^{j}}dz^{j} + i\left[\xi^{i}\frac{\partial^{2}L}{\partial z^{j}\partial \dot{z}^{i}} + \dot{\xi}^{i}\frac{\partial^{2}L}{\partial \dot{z}^{j}\partial \dot{z}^{i}}\right]d\dot{z}^{j} + \frac{\partial L}{\partial \dot{z}^{j}}d\dot{z}^{j} = B^{z}\sigma_{a}dz^{j} + B^{\dot{z}}\dot{\sigma}_{a}d\dot{z}^{j}$$

We infer the equations

$$\frac{\partial L}{\partial z^{i}} - i \frac{\partial}{\partial t} \left(\frac{\partial L}{\partial z^{i}} \right) = B^{z} \sigma_{a}, \frac{\partial L}{\partial \dot{z}^{i}} + i \frac{\partial}{\partial t} \left(\frac{\partial L}{\partial \dot{z}^{i}} \right) = B^{\dot{z}} \dot{\sigma}_{a}$$

Or

$$-i\left[\eta^{j}\frac{\partial^{2}L}{\partial x^{j}\partial x^{i}}+\dot{\eta}^{i}\frac{\partial^{2}L}{\partial \dot{x}^{j}\partial x^{i}}\right]dx^{j}+\frac{\partial L}{\partial x^{j}}dx^{j}+i\left[\eta^{i}\frac{\partial^{2}L}{\partial x^{j}\partial \dot{x}^{i}}+\dot{\eta}^{i}\frac{\partial^{2}L}{\partial \dot{x}^{j}\partial \dot{x}^{i}}\right]d\dot{x}^{j}+\frac{\partial L}{\partial \dot{x}^{j}}d\dot{x}^{j}=B^{x}\sigma_{a}dx^{j}+B^{\dot{x}}\dot{\sigma}_{a}d\dot{x}^{j}$$

we infer the equations

$$\frac{\partial L}{\partial x^{i}} - i \frac{\partial}{\partial t} \left(\frac{\partial L}{\partial x^{i}} \right) = B^{x} \sigma_{a}, \frac{\partial L}{\partial \dot{x}^{i}} + i \frac{\partial}{\partial t} \left(\frac{\partial L}{\partial \dot{x}^{i}} \right) = B^{\dot{x}} \dot{\sigma}_{a}$$

Or

$$-i\left[\zeta^{j}\frac{\partial^{2}L}{\partial y^{j}\partial y^{i}}+\dot{\zeta}^{i}\frac{\partial^{2}L}{\partial \dot{y}^{j}\partial y^{i}}\right]dy^{j}+\frac{\partial L}{\partial y^{j}}dy^{j}+i\left[\zeta^{i}\frac{\partial^{2}L}{\partial y^{j}\partial \dot{y}^{i}}+\dot{\zeta}^{i}\frac{\partial^{2}L}{\partial \dot{y}^{j}\partial \dot{y}^{i}}\right]d\dot{y}^{j}+\frac{\partial L}{\partial \dot{y}^{j}}d\dot{y}^{j}=B^{y}\sigma_{a}dy^{j}+B^{\dot{y}}\dot{\sigma}_{a}d\dot{y}^{j}$$

we infer the equations

$$\frac{\partial L}{\partial y^{i}} - i \frac{\partial}{\partial t} \left(\frac{\partial L}{\partial y^{i}} \right) = B^{y} \sigma_{a}, \frac{\partial L}{\partial \dot{y}^{i}} + i \frac{\partial}{\partial t} \left(\frac{\partial L}{\partial \dot{y}^{i}} \right) = B^{\dot{y}} \dot{\sigma}_{a}$$

Thus

$$\begin{aligned} \frac{\partial L}{\partial z^{i}} &- i \frac{\partial}{\partial t} \left(\frac{\partial L}{\partial z^{i}} \right) = B^{z} \sigma_{a}, \frac{\partial L}{\partial \dot{z}^{i}} + i \frac{\partial}{\partial t} \left(\frac{\partial L}{\partial \dot{z}^{i}} \right) = B^{\dot{z}} \dot{\sigma}_{a} \\ \frac{\partial L}{\partial x^{i}} &- i \frac{\partial}{\partial t} \left(\frac{\partial L}{\partial x^{i}} \right) = B^{x} \sigma_{a}, \frac{\partial L}{\partial \dot{x}^{i}} + i \frac{\partial}{\partial t} \left(\frac{\partial L}{\partial \dot{x}^{i}} \right) = B^{\dot{x}} \dot{\sigma}_{a} \end{aligned}$$

$$\frac{\partial L}{\partial y^{i}} - i \frac{\partial}{\partial t} \left(\frac{\partial L}{\partial y^{i}} \right) = B^{y} \sigma_{a}, \frac{\partial L}{\partial \dot{y}^{i}} + i \frac{\partial}{\partial t} \left(\frac{\partial L}{\partial \dot{y}^{i}} \right) = B^{\dot{y}} \dot{\sigma}_{a}$$
(23)

Thus, by complex Euler-Lagrange equations with constraints, we may call the equations obtained in (23) on $J^{(2,0)}\mathcal{M}$. Then the quartet $(J^{(2,0)}\mathcal{M}, \phi_L, \xi, \sigma)$ is named mechanical system with

5. Conclusions

From the study, we obtain that Lagrangian formalisms in generalized Classical Mechanics and field theory can be intrinsically characterized on $(J^{(2,0)}\mathcal{M}, \varphi_L, \xi)$ being a model of (2,0)-jet Bundles. So, the paths of semispray ξ on $J^{(2,0)}\mathcal{M}$ are the solutions of the Euler-Lagrange equations given by (11) on the mechanical system $(J^{(2,0)}\mathcal{M}, \varphi_L, \xi)$. Also, the solutions of the Euler Lagrange equations on (2,0)-jet Bundles with Constraints determined by (20) on the mechanical system $(J^{(2,0)}\mathcal{M}, \varphi_L, \xi, \sigma)$ are the paths of vector field ξ on $J^{(2,0)}\mathcal{M}$.

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