# **Research on Nonlinear Vibration Characteristics of Saddle- Shaped Orthotropic Membrane**

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#### Abstract

This paper, the nonlinear free vibration of saddle shaped Orthotropic Membrane is investigated. The Krylov-Bogolubov-Mitropolsky (KBM) perturbation method is employed for solving the governing equations of large amplitude nonlinear vibration of the membranes Presented herein are asymptotic analytical solutions for the frequency function of large amplitude nonlinear damped vibration of rectangular orthotropic membranes with four edges fixed. Through the computational example, The influence regularity of vibratory parameters such as structural parameters, initial displacement and vibration modes was studied. Which shows that the orthotropy and geometrical nonlinearity is significant for preventing destructive in membrane structures. In addition, the results provide some computational basis for the vibration control and dynamic design of membrane structures.

#### Keywords

Membrane Structures, KBM Perturbation Method, Nonlinear Vibration, Rise Span Ratio

#### 1. Introduction

Membrane structure is a new kind of tension structure carried load within membrane materials which have grown rapidly from the 1970s and now widely applied in large span structures. Because of its low weight, large flexibility, small damp and low natural vibration frequency, it is susceptible to vibration and relaxation deformation. As a result it will easily vibrate and the construction will be destroyed. Therefore, it is necessary to study the vibration characteristics of membrane structure to ensure a safe design.

In recent years more and more attention is being focused on the dynamic characteristics of membranes and many scholars have studied the free vibration theories of membranes but was limited to regular shape and The orthotropic properties of the membrane are not considered. Their researches involved the problems of: free vibration of confocal composite elliptical membranes [2]; free vibration of composite rectangular membranes [3, 4]; vibration of circular membranes [5, 6]; Free Vibration of Annular membrane [7]. a few articles discussed the orthotropic properties of the membrane in vibrating.

Using a semi-analytical method, the free vibration of saddle shaped orthotropic membranes is investigated in this paper, On the basis of large amplitude theory and D'Alembert's principle, the governing equation is addressed. Because damping exists in the governing equations, the solution of the governing equations will not be a periodic solution. Therefore, we apply the Krylov-Bogolubov-Mitropolsky (KBM) perturbation method to solve the nonlinear, damped, large amplitude vibration problem of orthotropic membranes with four fixed edges. computational examples are given to analyze the natural frequency change rules affected by each parameter. Some conclusions are also presented.

## 2. Governing Equations and Boundary Conditions

The saddle shaped Orthotropic Membrane model studied is orthotropic, with differential Young's moduli in its two principal fiber directions. Assume that the two principal fiber directions are just along with the two orthogonal directions x and y in a three-dimensional (3D) Cartesian coordinate system; see Fig. 1. The four edges of the model are embedded in an otherwise immovable structure. The spans in x and y are denoted by a and b, respectively; N0x and N0y denote the initial stress in x and y, respectively. (Point O0 is in the plane xoy.)



Figure 1. Saddle shaped membrane structure with four edges fixed.

For any saddle shaped Membrane model, the initial surface function  $z_0$  is:

$$z_0(x, y) = -\frac{4f_x}{a^2}x^2 + \frac{4f_y}{b^2}y^2$$
(1)

Where,  $f_x$  denote mid span arch in x,  $f_y$  denote mid span sag in y, and a and b denote the length of x and y direction.

The initial principal curvatures in x and y are:

$$k_{0x} = \frac{\partial^2 z_0(x, y)}{\partial x^2} = -\frac{8f_x}{a^2}, \ k_{0y} = \frac{\partial^2 z_0(x, y)}{\partial y^2} = \frac{8f_y}{b^2}$$
(2)

While the membrane is vibrating, the effect of shearing stress is so small that we may assume that  $N_{xy}=0$  in order to simplify the computation. According to the large amplitude theory and D'Alembert's principle [2], the dynamic equilibrium equation and the compatible equation of orthotropic membrane are

$$\left(N_{0y} + N_{y}\right)\left(k_{0y} + \frac{\partial^{2}w}{\partial y^{2}}\right) + \left(N_{0x} + N_{x}\right)\left(k_{0x} + \frac{\partial^{2}w}{\partial x^{2}}\right) = \rho \frac{\partial^{2}w}{\partial t^{2}} + c \frac{\partial w}{\partial t}$$

$$\frac{1}{E_{1}h} \frac{\partial^{2}N_{x}}{\partial y^{2}} + \frac{1}{E_{2}h} \frac{\partial^{2}N_{y}}{\partial x^{2}} - \frac{\mu_{1}}{E_{1}h} \frac{\partial^{2}N_{x}}{\partial x^{2}} - \frac{\mu_{2}}{E_{2}h} \frac{\partial^{2}N_{y}}{\partial y^{2}} = \left(\frac{\partial^{2}w}{\partial x \partial y}\right)^{2}$$

$$\left(\frac{\partial^{2}w}{\partial x^{2}} \frac{\partial^{2}w}{\partial y^{2}} - k_{x} \frac{\partial^{2}w}{\partial y^{2}} - k_{y} \frac{\partial^{2}w}{\partial x^{2}}\right)$$

$$\left(\frac{\partial^{2}w}{\partial x^{2}} + \frac{\partial^{2}w}{\partial y^{2}} + \frac{\partial^{2}w}{\partial y^{2}} + \frac{\partial^{2}w}{\partial x^{2}}\right)$$

$$\left(\frac{\partial^{2}w}{\partial x^{2}} + \frac{\partial^{2}w}{\partial y^{2}} + \frac{\partial^{2}w}{\partial y^{2}} + \frac{\partial^{2}w}{\partial x^{2}}\right)$$

Where  $\rho$  is the aerial density of membrane, *c* is the viscous damping;  $N_x$  and  $N_y$  denote additional tension in *x* and *y*, respectively;  $N_{0x}$  and  $N_{0y}$  denote initial tension in *x* and *y*, respectively; *w* denotes deflection: w(x, y, t); h denotes membrane's thickness;  $E_1$  and  $E_2$  denote Young's modulus in *x* and *y*, respectively;  $\mu_1$  and  $\mu_2$  denote Poisson's ratio in *x* and *y*, respectively.

The principal curvatures in *x* and *y* are:

$$k_x = k_{0x} + \Delta k_x, \ \Delta k_x = \frac{\partial^2 w}{\partial x^2}, \ k_y = k_{0y} + \Delta k_y, \ \Delta k_y = \frac{\partial^2 w}{\partial y^2}$$
(4)

Where  $\Delta k_x$  and  $\Delta k_y$  are principal curvature increments in x and y, respectively. The initial surface function Z(x, y, t) during the membrane's vibration is

$$z(x, y, t) = z_0(x, y) + w(x, y, t)$$
(5)

The corresponding curvatures in *x* and *y* can be expressed as:

$$k_x = -\frac{8f_x}{a^2} + \frac{\partial^2 w}{\partial x^2}, \ k_y = \frac{8f_y}{b^2} + \frac{\partial^2 w}{\partial y^2}$$
(6)

Compared with  $k_{0x}$  and  $k_{0y}$ , the higher order traces  $\Delta k_x$  and  $\Delta k_y$  can be ignored in the compatible equation.

Introducing the stress function  $\varphi(x, y, t)$  and letting:

$$N_x = h \frac{\partial^2 \varphi}{\partial y^2}, \quad N_y = h \frac{\partial^2 \varphi}{\partial x^2} \tag{7}$$

Equation (3) can be simplied as follows:

$$\begin{cases} \left( N_{0y} + h \frac{\partial^2 \varphi}{\partial x^2} \right) (k_{0y} + \frac{\partial^2 w}{\partial y^2}) + \left( N_{0x} + h \frac{\partial^2 \varphi}{\partial y^2} \right) (k_{0x} + \frac{\partial^2 w}{\partial x^2}) = \rho \frac{\partial^2 w}{\partial t^2} + c \frac{\partial w}{\partial t} \\ \frac{1}{E_1} \frac{\partial^4 \varphi}{\partial y^4} + \frac{1}{E_2} \frac{\partial^4 \varphi}{\partial x^4} = \left( \frac{\partial^2 w}{\partial x \partial y} \right)^2 - \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} - \left( k_{0x} \frac{\partial^2 w}{\partial y^2} + k_{0y} \frac{\partial^2 w}{\partial x^2} \right) \end{cases}$$
(8)

Where  $\varphi$  is the stress function  $\varphi(x, y, t)$ .

The corresponding boundary conditions may be expressed as:

$$\begin{cases} w(0, y, t) = 0 \\ w(a, y, t) = 0 \end{cases}, \begin{cases} w(x, 0, t) = 0 \\ w(x, b, t) = 0 \end{cases}$$
(9)

Under the action of  $N_{0x}$  and  $N_{0y},$  the equilibrium equation is obtained

$$k_{0x}N_{0x} + k_{0y}N_{0y} = 0 aga{10}$$

Substituting equation (10) into equation (8), yields:

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$$\int_{\partial x} h \frac{\partial^2 \varphi}{\partial y^2} + k_{0y} h \frac{\partial^2 \varphi}{\partial x^2} + (h \frac{\partial^2 \varphi}{\partial y^2} + N_{0x}) \frac{\partial^2 w}{\partial x^2} + (h \frac{\partial^2 \varphi}{\partial x^2} + N_{0y}) \frac{\partial^2 w}{\partial y^2} = \rho \frac{\partial^2 w}{\partial t^2} + c \frac{\partial w}{\partial t}$$
(11)

$$\frac{1}{E_1}\frac{\partial^4\varphi}{\partial y^4} + \frac{1}{E_2}\frac{\partial^4\varphi}{\partial x^4} = \left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 - \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - \left(k_{0x}\frac{\partial^2 w}{\partial y^2} + k_{0y}\frac{\partial^2 w}{\partial x^2}\right) \quad (12)$$

#### **3. Solution of Fundamental Equations**

The functions that satisfy the boundary conditions (9) are as follows:

$$\begin{cases} w(x, y, t) = W(x, y)T(t) \\ \varphi(x, y, t) = \phi(x, y)\tilde{T}(t) \end{cases}$$
(13)

Where W(x, y) is the given mode shape function,  $\phi(x, y)$ 

is the coordinate stress function; T(t) and  $\tilde{T}(t)$  are the unknown functions changed with time which reflects the vibration regularity. Assume that the mode shape function is given by

$$W(x, y) = \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}$$
(14)

Where m and n are integers, and denote the sine half-wave number in x and y directions. Equation (14) satisfies the boundary conditions automatically.

Substituting equation (14) into compatible equation (12), yields:

$$\frac{1}{E_1}\frac{\partial^4\varphi}{\partial y^4} + \frac{1}{E_2}\frac{\partial^4\varphi}{\partial x^4} = \frac{m^2n^2\pi^4}{2a^2b^2}T^2(t)\left(\cos\frac{2m\pi x}{a} + \cos\frac{2n\pi y}{b}\right) + \left(k_{0x}\frac{n^2\pi^2}{b^2} + k_{0y}\frac{m^2\pi^2}{a^2}\right)T(t)\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}$$
(15)

Assume that the stress function which satisfied equation. (15) is:

$$\begin{cases} \varphi(x, y, t) = T^{2}(t)\phi_{1}(x, y) + T(t)\phi_{2}(x, y) \\ \phi_{1}(x, y) = \left(\alpha \cos \frac{2m\pi x}{a} + \beta \cos \frac{2n\pi y}{b}\right); \\ \phi_{2}(x, y) = \delta \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} = \delta W(x, y) \end{cases}$$
(16)

Substituting equation (16) into equation (15), yields:

$$\alpha = \frac{E_2 a^2 n^2}{32b^2 m^2}, \quad \beta = \frac{E_1 b^2 m^2}{32a^2 n^2}, \quad \delta = \frac{k_{0x} (n\pi/b)^2 + k_{0y} (m\pi/a)^2}{(n\pi/b)^4 / E_1 + (m\pi/a)^4 / E_2}$$
(17)

Substituting equation (14), equation (16) and equation (17) into equation (11), and in view of the Bubnov-Galerkin method, yields:

$$\iint_{s} \begin{pmatrix} \rho \frac{\partial^{2} w}{\partial t^{2}} + c \frac{\partial w}{\partial t} - k_{0x} h \frac{\partial^{2} \varphi}{\partial y^{2}} - k_{0y} h \frac{\partial^{2} \varphi}{\partial x^{2}} - \\ (h \frac{\partial^{2} \varphi}{\partial y^{2}} + N_{0x}) \frac{\partial^{2} w}{\partial x^{2}} - (h \frac{\partial^{2} \varphi}{\partial x^{2}} + N_{0y}) \frac{\partial^{2} w}{\partial y^{2}} \end{pmatrix}^{W}(x, y) ds$$

$$= \iint_{s} \begin{pmatrix} \rho W \frac{\partial^{2} T(t)}{\partial t^{2}} + c W \frac{\partial T(t)}{\partial t} - \begin{pmatrix} k_{0x} h \frac{\partial^{2} \phi}{\partial y^{2}} + k_{0y} h \frac{\partial^{2} \psi}{\partial x^{2}} + \\ N_{0x} \frac{\partial^{2} W}{\partial x^{2}} + N_{0y} \frac{\partial^{2} W}{\partial y^{2}} \end{pmatrix}^{T}(t)$$

$$= \iint_{s} \begin{pmatrix} -h \left( k_{0x} \frac{\partial^{2} \phi}{\partial y^{2}} + k_{0y} \frac{\partial^{2} \phi}{\partial x^{2}} + \frac{\partial^{2} \phi}{\partial y^{2}} \frac{\partial^{2} W}{\partial x^{2}} + \frac{\partial^{2} \phi}{\partial x^{2}} \frac{\partial^{2} W}{\partial y^{2}} \end{pmatrix} T^{3}(t)$$

$$= 0$$

Eq. (18) can be simplified as

$$A\frac{\partial^2 T(t)}{\partial t^2} + B\frac{\partial T(t)}{\partial t} + CT(t) + DT^2(t) + ET^3(t) = 0$$
(19)

Where:

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$$A = \iint_{S} \rho W^{2} dx dy = \iint_{S} \rho \sin^{2} \frac{m\pi x}{a} \sin^{2} \frac{n\pi y}{b} ds = \frac{\rho ab}{4}$$

$$B = \iint_{S} cW^{2} dx dy = \iint_{S} c \sin^{2} \frac{m\pi x}{a} \sin^{2} \frac{n\pi y}{b} ds = \frac{cab}{4}$$

$$F = -\iint_{S} \left( k_{0x} h \frac{\partial^{2} \phi_{2}}{\partial y^{2}} + k_{0y} h \frac{\partial^{2} \phi_{2}}{\partial x^{2}} + N_{0x} \frac{\partial^{2} W}{\partial x^{2}} + N_{0y} \frac{\partial^{2} W}{\partial y^{2}} \right) W ds$$

$$-\iint_{S} \left[ \left( hk_{0x} \delta + N_{0y} \right) \frac{\partial^{2} W}{\partial y^{2}} + \left( hk_{0y} \delta + N_{0x} \right) \frac{\partial^{2} W}{\partial x^{2}} \right] W ds$$

$$\frac{m^{2} \pi^{2} b^{2} (hk_{0y} \delta + N_{0x}) + n^{2} \pi^{2} a^{2} (hk_{0x} \delta + N_{0y})}{4ab}$$

$$D = -\iint_{S} h \left( k_{0x} \frac{\partial^{2} \phi}{\partial y^{2}} + k_{0y} \frac{\partial^{2} \phi}{\partial x^{2}} + \frac{\partial^{2} \phi}{\partial y^{2}} \frac{\partial^{2} W}{\partial x^{2}} + \frac{\partial^{2} \phi}{\partial x^{2}} \frac{\partial^{2} W}{\partial y^{2}} \right) W(x, y) ds$$

$$= -\frac{16n^{2} \pi^{2} h k_{0x} ab \beta}{3b^{2} m \pi^{2}} - \frac{16m^{2} \pi^{2} h k_{0y} ab \alpha}{3a^{2} m \pi^{2}} - \frac{32hm \pi^{2} \delta}{9ab}$$

$$E = \iint_{S} - \left( \frac{\partial^{2} \phi}{\partial y^{2}} \frac{\partial^{2} W}{\partial x^{2}} + \frac{\partial^{2} \phi}{\partial x^{2}} \frac{\partial^{2} W}{\partial y^{2}} \right) W ds$$

$$= \frac{3hm^{2} n^{2} \pi^{4} (\alpha + \beta)}{2ab}$$

Because there is damping in Eq. (19), its solution will not be a periodic solution. In addition, Eq. (19) is a nonlinear differential equation [1]. Therefore, it is very difficult to obtain its analytical solution. We apply the KBM perturbation method to obtain the approximate analytical solution that satisfies Eq. (19).

Assume that the perturbation parameter is 
$$\varepsilon = \frac{h^2}{ab} \ll 1$$
 and

let  $\chi = \chi(t) = T(t)$ . Equation (19) can then be simplified as

$$\frac{\partial^2 \chi}{\partial t^2} + \omega_0^2 \chi = \varepsilon f(\chi, \frac{d\chi}{dt}) = \varepsilon (\alpha_1 \chi^2 + \alpha_2 \chi^3 + \alpha_3 \frac{d\chi}{dt}) \quad (20)$$

Where:

$$\omega_{0}^{2} = \frac{m^{2}\pi^{2}b^{2}(hk_{0y}\delta + N_{0x}) + n^{2}\pi^{2}a^{2}(hk_{0x}\delta + N_{0y})}{\rho a^{2}b^{2}}$$
$$\alpha_{1} = \frac{192n^{2}k_{0x}a^{2}\beta + 192m^{2}k_{0y}b^{2}\alpha + 128m^{2}n^{2}\pi^{2}\delta}{9ab\rho mnh}$$
$$\alpha_{2} = -\frac{6m^{2}n^{2}\pi^{4}(\alpha + \beta)}{\rho h^{2}a^{2}b^{2}} \quad \alpha_{3} = -\frac{cab}{\rho h^{2}}$$

Assume that the solution of Eq. (16) is:

$$\chi = a \cos \psi \tag{21}$$

where a and  $\Psi$  are not constants since they are functions of time t. they can obtained by:

$$\begin{cases}
\frac{da}{dt} = -\frac{\varepsilon}{\omega_0} A_0(a) \\
\frac{d\psi}{dt} = \omega_0 - \frac{\varepsilon}{a\omega_0} C_0(a)
\end{cases}$$
(22)

In Eq. (22),

$$A_{0}(a) = \frac{1}{2\pi} \int_{0}^{2\pi} \sin \psi \cdot f(a \cos \psi, -a\omega_{0} \sin \psi) d\psi$$
  
$$= \frac{1}{2\pi} \int_{0}^{2\pi} \sin \psi \cdot (\alpha_{1}a^{2} \cos^{2}\psi + \alpha_{2}a^{3} \cos^{3}\psi - a\omega_{0}\alpha_{3} \sin \psi) d\psi \qquad (23)$$
  
$$= -0.5\alpha_{3}a\omega_{0}$$

$$C_{0}(a) = \frac{1}{2\pi} \int_{0}^{2\pi} \cos \psi \cdot f(a \cos \psi, -a\omega_{0} \sin \psi) d\psi$$
  
=  $\frac{1}{2\pi} \int_{0}^{2\pi} \cos \psi \cdot (\alpha_{1}a^{2} \cos^{2}\psi + \alpha_{2}a^{3} \cos^{3}\psi - a\omega_{0}\alpha_{3} \sin \psi) d\psi$  (24)  
=  $\frac{3\alpha_{2}a^{3}}{8}$ 

Substituting Eq. (23) and Eq. (24) into Eq. (22), yields:

$$\begin{cases} \frac{da}{dt} = \frac{\varepsilon \alpha_3 a}{2} \\ \frac{d\psi}{dt} = \omega_0 - \frac{\varepsilon}{a\omega_0} \frac{3\alpha_2 a^3}{8} \end{cases}$$
(25)

Integrating Eq. (25) and using the variable separation method, yields:

$$\begin{cases} a = a_0 e^{0.5\alpha_3 \varepsilon t} \\ \psi = (\omega_0 - \frac{3\alpha_2 a^2 \varepsilon}{8\omega_0})t + \psi_0 \end{cases}$$
(26)

Substituting Eq. (26) into Eq. (21), yields:

$$\chi = a_0 e^{0.5\alpha_3 \varepsilon t} \cos((\omega_0 - \frac{3\alpha_2 a_0^2 \varepsilon e^{\alpha_3 \varepsilon t}}{8\omega_0})t + \psi_0)$$
(27)

In Eq. (27),  $a_0$  is the amplitude and  $\Psi_0$  is the initial phase. They are determined by the initial conditions. Assume that the general initial conditions are:

$$\chi(t)\Big|_{t=0} = b_0, \frac{d\chi(t)}{dt}\Big|_{t=0} = v_0$$
 (28)

Where  $b_0$  and  $v_0$  denote the initial displacement and velocity of point  $(x_0, y_0)$ .

Taking the derivative of function (27), yields.

$$\frac{d\chi(t)}{dt} = \frac{a_0}{2} \alpha_3 e^{0.5\alpha_3 \epsilon t} \cos((\omega_0 - \frac{3\alpha_2 a_0^{-2} \epsilon e^{\alpha_3 \epsilon t}}{8\omega_0})t + \psi_0) - a_0 e^{0.5\alpha_3 \epsilon t} \sin((\omega_0 - \frac{3\alpha_2 a_0^{-2} \epsilon e^{\alpha_3 \epsilon t}}{8\omega_0})t + \psi_0) \cdot \left(\frac{\omega_0 - \frac{3\alpha_2 a_0^{-2} \epsilon e^{\alpha_3 \epsilon t}}{8\omega_0}}{\frac{3\alpha_2 \alpha_3 a_0^{-2} \epsilon^2 e^{\alpha_3 \epsilon t}}{8\omega_0}t}\right)$$
(29)

In order to simplify the calculations, we assume that the initial displacement is  $b_0 = a_0$  and the initial phase is  $\psi_0 = 0$ , and Eq. (27) can be transformed into:

$$\chi = T(t) = a_0 e^{0.5\alpha_3 \varepsilon t} \cos(\omega_0 - \frac{3\alpha_2 a_0^2 \varepsilon e^{\alpha_3 \varepsilon t}}{8\omega_0})t$$
(30)

By substituting Eqs. (14) and (30) into the first formula of Eq. (13), we can obtain the displacement function of nonlinear large amplitude vibration of membranes with viscous damping.

$$w(x, y, t) = \sin \frac{m\pi x}{a} \cdot \sin \frac{n\pi y}{b} \cdot (a_0 e^{0.5\alpha_3 \varepsilon t} \cos(\omega_0 - \frac{3\alpha_2 a_0^2 \varepsilon e^{\alpha_3 \varepsilon t}}{8\omega_0})t) \quad (31)$$

Equation (30) is the approximate analytical solution for Eq. (19). In view of Eq. (30), the approximate analytical expression of the frequency is:

$$\omega = \omega_0 - \frac{3\alpha_2 a_0^2 \varepsilon e^{\alpha_3 \varepsilon t}}{8\omega_0}$$
(32)

## 4. Computational Examples and Discussion

Considering the membrane material used in this project as an example, we have the Young's modulus in x and y as E1 =  $1.4 \times 106$  and E2 =  $0.9 \times 106$  kN/m, respectively; the aerial density of membranes is  $\rho = 1.7$  kg/m<sup>2</sup>; the membrane's thickness is h = 1.0 mm; The viscous damping is c =90 N s/m; ratio of span is  $\lambda = \frac{b}{a}$ ; ratio of arch to span in x and y are  $\varepsilon_x = \frac{f_x}{a}$  and  $\varepsilon_y = \frac{f_y}{b}$ , respectively.

# 4.1. Values of Frequency <sup>(a)</sup> (rad/s) Under Different Initial Displacements <sup>a</sup>0

We have the initial tension in x and y are  $N_{0x} = N_{0y} = 3KN$ . the length of x and y direction are a = b = 1m; ratio of arch to span in x and y are  $\varepsilon_x = \varepsilon_y = 0.1$ .

The curve of frequency (rad/s) under different initial displacements is shown in Fig. 2. (t = 0.01s)



Figure 2. Frequency (rad/s) under different initial displacements.

According to Fig. 2, Frequency values nonlinearly increase with respect to initial displacement and the larger the initial displacement, the faster the rate of increase in vibration frequency. Meanwhile, the frequency values also increase with increasing vibration order.



*Figure 3. Three-dimensional diagram of frequency (rad/s) and vibration order*  $(a_0 = 0.01m)$ *.* 



*Figure 4.* Three-dimensional diagram of frequency (rad/s) and vibration order ( $a_0 = 0.001m$ ).

#### 4.2. Values of Frequency $\omega$ (rad/s) Under Different Span Ratio $\lambda$

The curve of span ratio and vibration frequency is shown in Fig. 5. ( $N_{0x} = N_{0y} = 3KN$ ,  $\varepsilon_x = \varepsilon_y = 0.1$ , t = 0.01s,  $a_0 = 0.001m$ )



Figure 5. Curve of span ratio and vibration frequency.

According to Fig. 5, The increase of span ratio  $\lambda$  and the comparison of the different models show that Frequency values decreases with increasing span ratio  $\lambda$ . when  $\lambda \leq 0.2$ , the natural vibration frequency  $\omega$  in all models decrease sharply.  $\omega$  increases gently when  $\lambda \geq 0.2$ , which shows that the near-span sizes (a  $\approx$  b) should be avoided in saddle shaped structures. which shows that the span-ratio value should not be given too large in saddle shaped model a greater  $\omega$  can be obtained if the smaller modulus is arranged in the long-side direction when  $\lambda \neq 0.2$  (just as E1 > E2). The more

discrepant the two span sizes (b and a), the greater the vibration frequency; when  $\lambda$ =0.2, two frequency values are equal.

#### 4.3. Values of Frequency<sup>(i)</sup> (rad/s) Under Different Ratio of Arch-to-Span in x and y

The curve of arch-to-span ratio and vibration frequency is shown in Fig.6 and Fig.7. ( $N_{0x} = N_{0y} = 3KN$ , a = b = 1, t = 0.01s,  $a_0 = 0.001m$ )



Figure 6. Three-dimensional diagram of frequency (rad/s) and arch-to-span ratio.



Figure 7. Curve of frequency (rad/s) and arch-to-span ratio.

When the two sizes (sag  $f_x$  and arch  $f_y$ ) are close to one another, the increasing  $\varepsilon$  has little effect on the structural stability, The reasons may be that when they are the same in size, relative curvature of the structure in both directions will changes lowly. Structure can be treated as planar model at this point, the influences of arch-to-span ratio on the vibration frequency weaken gradually, which is a negligible factor in designing the kind of structure.

The natural of membrane is larger as the difference between ratio of arch-to-span in x and y increases.

#### 5. Conclusions

In this work, an analytical method was used to study the nonlinear vibration of saddle- shaped orthotropic membrane structures in the large amplitude theory. The influence of the initial structure parameter of membrane on vibrational frequency were analyzed and discussed. Based on the results obtained from the considered example, we can conclude that:

(1) The result indicates that with the Initial deflection values increasing, the values of frequency show non-linear growth. And the larger the initial displacement, the faster the rate of increase in vibration frequency.

(2) Frequency values decreases with increasing span ratio. when  $\lambda \leq 0.2$ , the natural vibration frequency in all models decrease sharply. vibration frequency increases gently when  $\lambda \geq 0.2$ , For the orthotropy of membrane, in the saddle shaped model a greater  $\omega$  can be obtained if the smaller modulus is arranged in the long-side direction.

(3) When the two sizes (sag  $f_x$  and arch  $f_y$ ) are close to one another, the increasing  $\varepsilon$  has little effect on the structural stability.

The results obtained herein provide a simple and convenient approach to understand the nonlinear vibration characteristics of saddle-shaped orthotropic membranes in dynamic environment. In addition, the results provide some computational basis for the vibration control and dynamic design of membrane structures.

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