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Abstract

This paper examines the theoretical implication of quantity-discounted transportation rates on the impact of a change market demand on the optimum location decisions of undifferentiated oligopolistic firms within the Weber-Moses triangle. Assume that transportation rates are a function of quantity shipped and distance traveled. We show that the optimum location is independent of a change in market demand if the market demand function is linear. This is consistent with the well-known Mai-Hwang results with constant transportation rates. It indicates that Mai and Hwang proposition is more general than it appears. We further show that an increase in market demand may move the optimum location closer to (away from) the output market when the market demand function is concave (convex). These results are significantly different from the well-known Mai-Hwang results with constant transportation rates. It indicates that the quantity discounted transportation rates have an important influence on the location decisions of undifferentiated oligopolistic firms.

Keywords

Undifferentiated Oligopoly, Plant Location, Quantity Discounted Transportation Rates, Weber-Moses Triangular Location Model

1. Introduction

Since Moses's path-breaking work [1], *Location and the Theory of Production*, most of the studies in location theory of firms has focused on two polar cases: perfect competition and monopoly. Little attention is devoted to the intermediate and more realistic cases: oligopoly and monopolistic competition. In a frequently cited paper, *Production-Location Decision and Free Entry Oligopoly*, Mai and Hwang [2] (henceforth MH) incorporated undifferentiated oligopolistic competition into the Moses-Weber triangular location model and attempted to fill this gap. Under the assumptions that (1) firms are symmetric and identical; (2) firms produce a homogenous good and make Cournot-Nash conjectures about their rivals' production and location decisions; (3) the production function exhibits increasing returns to scale¹; (4) firms are free to enter and leave the industry, they established the following interesting and important propositions.

MH1. The optimum location of an undifferentiated firm is independent of a change in demand if the demand function is linear.

MH2. The optimum location of an undifferentiated firm moves toward (away from) the output market as demand increases if the demand function is convex (concave). MH [2, pp.258-260]

These results crucially depend upon the constant transportation rates assumption. However, as is well known in transportation economics, discounts for quantity shipped and distance traveled are prevalent among various modes of transportation, cf. Fair and Williams [4, pp. 320-321, p. 325]. Meanwhile, Miller and Jensen [5], Shieh and Mai [6], Gilley, Shieh and Williams [7] and others have examined the theoretical implication of quantity and distance discounted transportation rates on the location decision of a perfectly

^{1.} MH [2] also considered the impact of demand on the location decision when the production function exhibits constant or decreasing returns to scale. However, it can be shown that no interior solution exists if there are constant or decreasing

returns to scale in production and free entry. In this note, we will consider the increasing returns to scale only. See also Hwang, Mai and Shieh [3].

competitive firm or a monopolistic firm.

Since MH's 1992 model serves as a basis for a recent attempt to integrate plant location with trade policy and domestic commodity taxes, cf. Huang and Mai [8], Chen and Shieh [9] and Shieh [10], it would be interesting and important to examine the impact of quantity-discounted transportation rates on output and the location decisions of oligopolistic firms. We first apply the conventional profit maximization approach to find the optimal output and plant location, and then utilize the comparative static analyses to investigate the effect of a change in market demand on the plant location decisions of undifferentiated oligopolistic firms. It will be shown that MH1 holds if market demand function is linear. However, MH2 may not hold if transportation rates are dependent of quantity shipped.

2. An Oligopolistic Location Model

Following MH [2] and Hwang, Mai & Shieh (2007), our analysis is based on the following assumptions.

(a) N firms employ two transportable inputs (L and K) located at A and B to produce a homogenous product (q) which is sold at the output market C. The location triangle in Figure 1 illustrates the location problem of oligopolistic firms. In Figure 1, the distance a and b and the angle $\pi/2 \ge \gamma \ge 0$ are known; h is the distance between the plant location (E) and C; s and z are the distances of plant location (E) from A and B, respectively; θ is the angle between CA and CE.



Figure 1. The Weber-Moses Triangle.

(b) Firms make Cournot-Nash conjectures about their rivals' production and location decisions and enter the industry without any restrictions until there is no economic profit. Assume also that equilibra are symmetric. Thus, we can neglect the location dispersion of firms and focus on the impact of market demand on the location decision of a representative firm.

(c) The production function is homogenous of degree n,

$$q = f(L, K) \tag{1}$$

with the following properties:

$$\begin{split} f_{LL} + f_{KK} &= nq, \\ f_{LL}L + f_{LK}K &= (n-1)f_L, \\ f_{KL}L + f_{KK}K &= (n-1)f_K, \\ f_{LL}L^2 + 2f_{LK}LK + f_{KK}K^2 &= n(n-1)q \end{split}$$

where $f_L \equiv \partial q/\partial L > 0$, $f_K \equiv \partial q/\partial K > 0$, $f_{LK} \equiv f_{KL} \equiv \partial q^2/\partial L\partial K > 0$, $f_{LL} \equiv \partial^2 q/\partial L^2 < 0$, $f_{KK} \equiv \partial^2 q/\partial K^2 < 0$. MH [2] assumes that the production function is homothetic. To simplify our analysis and make calculation tractable, we assume that the production function is homogenous of degree n.

(d) The industry inverse demand function for output is given by

$$P = P(Q, \alpha) \tag{2}$$

where $Q = \sum q^i$ is the market quantity demanded, $P_Q \equiv \partial P / \partial Q$ < 0, $P_\alpha \equiv \partial P / \partial \alpha > 0$ and $P_{Q\alpha} \equiv \partial P^2 / \partial Q \partial \alpha = 0$, cf. MH [2, p. 256]. It should be noted that

 \sum denotes $\sum_{i=1}^{N}$.

(e) The prices of inputs and output are evaluated at the plant location (E). The cost of purchasing inputs is the price of input at the source plus the freight cost, and the price of output is the market price minus the freight cost.

(f) The transportation rates for inputs and output are specified as:

$$k = k(s, L), m = m(z, K) \text{ and } t = t(h, q)$$
 (3)

where k, m and t are transportation rates of L, K, and q; s, z and h are distances from the plant location to the sources A, B and the output market C. $k_s \equiv \partial k/\partial s < 0$ $m_z \equiv \partial m/\partial z < 0$ $t_h \equiv \partial t/\partial h < 0$ $k_L \equiv \partial k/\partial L < 0$ $m_K \equiv \partial m/\partial K < 0$ $t_q \equiv \partial t/\partial q < 0$. By the law of cosines, we can express s and z as:

$$s = (a^{2} + h^{2} - 2ahcos\theta)^{1/2}, z = [b^{2} + h^{2} - 2bhcos(\gamma - \theta)]^{1/2}$$
(4)

(g) The objective of each firm is to find the optimum location and production within the Weber- Moses triangle which maximizes the profit.

It should be noted that the inclusion of quantity shipped and distance traveled as key variables in transportation rate functions constitutes the only point of departure from MH's oligopolistic location model.

With these assumptions, the profit maximizing location problem of the representative firm is given by

Max
$$\Pi = [P(Q, \alpha)-t(h,q)h]q - [w+k(s,L)s]L-[r+m(z,K)z]K$$
 (5)

where q, L, K, h, and θ are decision variables. Assuming that the profit maximizing oligopolistic firm treats the output q instead of L and K as a decision variable, we first deal with the following constrained production cost minimization problem at a given plant location.

Min
$$L = [w+k(s,L)s]L + [r+m(z,K)z]K + \lambda[q - f(L, K)]$$
 (6)

where λ is the Lagrange multiplier. Setting the partial

derivatives of L with respect to L, K and λ equal to zero yields the first-order conditions for a minimum:

$$(\partial L/\partial L) = w + ksu_L - \lambda f_L = 0$$
⁽⁷⁾

$$(\partial L/\partial \mathbf{K}) = \mathbf{r} + \mathbf{m}\mathbf{z}\mathbf{u}_{\mathbf{K}} - \lambda \mathbf{f}_{\mathbf{K}} = 0 \tag{8}$$

$$(\partial L/\partial \lambda) = q - f(L,K) = 0$$
(9)

where $u_L \equiv 1 - c_L$, $u_k = 1 - c_K$, $c_L \equiv -(\partial k/\partial L)(L/k)$, $c_K \equiv -(\partial m/\partial K)(K/m)$. c_L and c_K are the elasticities of transportation rate with respect to L and K, respectively. To ease our analysis, we assume that c_L and c_K are constant and $u_L > 0$, $u_K > 0$, cf. Miller and Jensen [5], Shieh and Mai [6] Gilley, Shieh and Willians [7] and Shieh and Yeh [11].

If the second-order conditions are satisfied, (7) - (9) can be solved for L and K in terms of q. The relationship between L (or K) and q can be derived by applying the standard comparative static analysis, (see Appendix).

$$dL/dq = (1/D)(Af_{KK} - Bf_{LK} - m_K z u_K f_L)$$
(10)

$$dK/dq = (1/D)(Bf_{LL} - Af_{KL} - k_L su_L f_K)$$
(11)

where

$$D = (1/L)[nq(Af_{KK} - Bf_{LK}) - (f_{K}^{2}k_{L}su_{L} + f_{L}^{2}m_{k}zu_{K})L]$$

= (1/K)[nq(Bf_{LL} - Af_{KL}) - (f_{K}^{2}k_{L}su_{L} + f_{L}^{2}m_{k}zu_{K})K] (12)

and $A \equiv w + ksu_L$, $B \equiv r + mzu_K$, $k_L \equiv \partial k / \partial L$, $m_K \equiv \partial m / \partial K$.

In the case where transportation rates are constant, (i.e., MH's case), or a function of distance traveled only, i.e., $c_L = c_K = 0$, we obtain

$$dL/dq = (L/nq)$$
 and $dK/dq = (K/nq)$ (13)

Equation (13) is identical to the conventional result in the non-spatial model, cf. Ferguson [12, pp. 142-143] and Silberberg and Suen [13, pp. 205-206]. If the production function is homogenous and delivered prices are independent of input usage, input proportion depends only upon the constant delivered price ratio, and a change in output will not change input proportion. The expansion path is a ray through the origin in (L, K) dimension. However, in the case where transportation rates are a function of quantity shipped, the delivered price ratio changes with input usage. A change in output and input usage will change the delivered price ratio and then input proportion. Thus, the expansion path is not an isocline. The result in (13) does not hold.

It is of interest to note that if transportation rates are constant, as MH [2, p. 225] points out, the production cost function can be written as the product of two functions: a function of delivered prices only and another function of output, i.e., C(q) = c(w + ks, r + mz)H(q). However, if transportation rates are a function of quantity shipped and distance traveled, the delivered prices are a function of output, the production cost would be $C(q) = \{w + k[L(q),s]s\}L(q) + \{r + m[K(q), z]z\}K(q)$.

Substituting the input demand functions of L and K in terms of q into (5), we obtain the profit as a function of q, h

and θ . The first-order conditions for a maximum are

$$(\partial \pi/\partial q) = (P+P_Qq) - thu - (w+ksu_L) (dL/dq) - (r+mzu_K) (dK/dq) = 0$$
(14)

$$(\partial \pi / \partial h) = -tqv - ks_h v_L L - mz_h v_K K = 0$$
(15)

$$(\partial \pi / \partial \theta) = -ks_{\theta}v_{L}L - mz_{\theta}v_{K}K = 0$$
 (16)

where $s_h \equiv \partial s/\partial h$, $z_h \equiv \partial z/\partial h$, $s_\theta \equiv \partial s/\partial \theta$, $z_\theta \equiv \partial z/\partial \theta$, u = 1 - c, $c \equiv -(\partial t/\partial q)(q/t)$, v = 1 - d, $d \equiv -(\partial t/\partial h)(h/t)$, $v_L = 1 - d_L$, $d_L \equiv -(\partial k/\partial s)(s/k)$, $v_K = 1 - d_K$, and $d_K \equiv -(\partial m/\partial z)(z/m)$. c is the elasticity of transportation rate with respect to output shipped. d, d_L , d_K are the elasticities of transportation rates with respect to distances, h, s and z respectively. For simplicity, we assume that u, v, v_L and v_K are positive constant throughout the paper. We further assume that the secondorder conditions are satisfied and the possibility of a corner solution is excluded, cf. Miller and Jensen [5], Kusumoto [14] and MH [2]. Thus, we can solve (14) – (16) for q, h and θ when free entry is prohibited, i.e., the number of firms (N) is given.

If free entry is allowed, each firm in the industry earns normal profit only. The following condition must be satisfied.

$$\pi = [P(Nq, \alpha) - t(h,q)h]q - \{w+k[s,L(q)s]\}L(q) + \{r+m[z,K(q)]z\}K(q) = 0$$
(17)

If there is an interior solution, we can solve (14) - (17) for q, h, θ and N in terms of α and e = (a, b, γ , w, r), where e is a vector of remaining parameters.

$$q = q(\alpha, e), h = h(\alpha, e), \theta = \theta(\alpha, e), N = N(\alpha, e)$$
(18)

The expressions for the partial derivates such as $\partial q/\partial \alpha$, $\partial h/\partial \alpha$, $\partial \theta/\partial \alpha$ and $\partial N/\partial \alpha$ can be obtained by applying the standard comparative static analyses. This completes our modeling of the basic analytical framework for studying of the effects of a change in demand on the oligopolistic firm's production and location decisions.

3. Effect of Demand on Location Decision

We are now in a position to investigate the effect of a change in demand on the optimum location. Totally differentiating (14) - (17) and applying Cramer's rule, we obtain

$$(\partial h/\partial \alpha) = (1/D_4) P_{\alpha} q^3 P_{QQ}(\pi_{\theta q} \pi_{\theta h} - \pi_{\theta \theta} \pi_{hq})$$
(19)

$$(\partial \theta / \partial \alpha) = (1/D_4) P_{\alpha} q^3 P_{QQ}(\pi_{\theta h} \pi_{hq} - \pi_{\theta q} \pi_{hh})$$
(20)

where

$$\pi_{\theta q} = -ks_{\theta}v_{L}u_{L}(dL/dq) - mz_{\theta}v_{K}u_{K}(dK/dq)$$

$$\pi_{\theta h} = -(d_{L}/s)ks_{h}s_{\theta}v_{L}L - (d_{K}/z)mz_{h}z_{\theta}v_{K}K - ks_{\theta h}v_{L}L - mz_{\theta h}v_{K}K$$

$$\pi_{\theta \theta} = -(d_{L}/s)ks_{\theta}^{2}v_{L}L - (d_{K}/z)mz_{\theta}^{2}v_{K}K - ks_{\theta \theta}v_{L}L - mz_{\theta \theta}v_{K}K$$

$$\pi_{hq} = -tvu - ks_{h}v_{L}u_{L}(dL/dq) - mz_{h}v_{K}u_{K}(dK/dq)$$

$$\pi_{hh} = -(d/h)tqv - (d_L/s)ks_h^2 v_L L - (d_K/z)mz_h^2 v_K K - ks_{hh}v_L L$$

$$-mz_{hh}v_KK$$

 $D_4 = P_Q q^2 D_3 - q [P_Q q (N-1)] (P_Q + P_{QQ} q) D_2$

 D_2 and D_3 are the second-order and third-order principal minor of Hessian determinant $D_4.$ $D_4 > 0,$ $\pi_{\theta\theta} < 0$ and $\pi_{hh} < 0$ are based on the second-order conditions and the stability condition.

Assume that the market demand function is linear, i.e., $P_{QQ} = 0$. From (19) and (20), we at once obtain $(\partial h/\partial \alpha) = 0$ and $(\partial \theta/\partial \alpha) = 0$. Thus, we conclude that

Proposition 1. With free entry, when transportation rates are a function of quantity shipped and distance traveled, the optimum location is independent of a change in demand if the market demand function is linear.

This result is consistent with MH1 in the constant transportation rates case. However, we show that MH1 is more general than it appears. The economic interpretation behind this proposition is given as follows. It is well known that in the Weber-Moses location theory the optimum location is determined by the relative strength of the market pull and two material pulls. In the case where the market demand function is linear, a change in demand will not change output per firm and usage of L and K. In this case the market pull and the material pulls remain the same. As a result, the optimum location is invariant.

In the case where the demand function is not linear, i.e., $P_{QQ} \neq 0$, we consider two specific situations regarding transportation rates: (1) transportation rates are dependent of distance traveled but independent of quantity shipped; (2) transportation rates are dependent of quantity shipped and distance traveled.

3.1. Transportation Rates are Independent of Quantity Shipped

In this case, $u = u_L = u_K = 1$ and dL/dq = L/nq, dK/dq = K/nq. Using the first-order conditions in (15) and (16), we can rewrite (19) and (20) as:

 $(\partial h/\partial \alpha) = (1/D_4) P_{\alpha} q^3 P_{QQ} \pi_{\theta \theta}(v/n) t(n-1)$ (19a)

$$(\partial \theta / \partial \alpha) = (1/D_4) P_{\alpha} q^3 P_{OO} \pi_{\theta h} (v/n) t(n-1)$$
(20a)

Since $D_4 > 0$, $\pi_{\theta\theta} < 0$, n > 1, and the sign of $\pi_{\theta h}$ is, a priori, not certain, the sign of $(\partial \theta / \partial \alpha)$ is ambiguous. Thus, the effect of a demand change on the circumferential location is ambiguous. However, we can show $(\partial h / \partial \alpha) > (<) 0$ as $P_{QQ} < (>) 0$. Thus, we can conclude that

Proposition 2. With free entry, when transportation rates are a function of distance traveled, the optimum location moves toward (away from) the output market as demand increases if the market demand function is convex (concave).

This result shows that MH2 can be applied to the case where the transportation rates are a function of distance traveled only. The economic interpretation is given as follows. When the market demand function is convex, an increase in demand causes each firm's output increases. Since the production exhibits increasing returns to scale, the quantities of the inputs per unit of output decrease, then the material pulls decrease and the market pull increases. As a result, the optimum location moves toward the output market.

3.2. Transportation Rates are Dependent of Quantity Shipped

In this case, $u \neq u_L \neq u_K \neq 1$ and $v \neq v_L \neq v_K \neq 1$. Combining (10), (11) and (19), (20), we can see that the signs of $(\partial h/\partial \alpha)$ and $(\partial \theta/\partial \alpha)$ are ambiguous. Thus, we can conclude that

Proposition 3. With free entry, when transportation rates are a function of quantity shipped, the optimum location need not move toward (away from) the output market as demand increases even if the market demand function is convex (concave).

This result is significantly different from MH2. It is well known that in the Weber-Moses location theory the optimum location is determined by the relative strength of the market pull and two material pulls. Each pull is comprised of the quantity and marginal transport cost components. In the case where the production function exhibits increasing returns to scale and transportation rates are independent of quantity shipped, MH shows that if the market demand is convex, an increase in demand causes each firm's output to rise and the input-output ratio will fall. The importance of material pulls relative to the market pull decreases. As a result, the optimum location moves toward the output market. If transportation rates depend upon quantity shipped, the importance of material pulls relative to the market pull may not decrease. Thus, the impact of a change in demand on the optimum location decision is unpredictable.

4. Conclusion

We have introduced quantity shipped and distance traveled into transportation rate functions and examined the impact of these variables on the location decisions of undifferentiated oligopolistic firms with free entry. Under the assumptions that transportation rates are constant and the production function exhibits increasing returns to scale, MH [2] showed that (1) the optimum location is independent of a change in demand if the demand function is linear; (2) the optimum location will move toward (away from) the output market as demand increases if the market demand function is convex (concave).

When transportation rates are a function of distance traveled only, we show that MH1 and MH2 hold. This indicates that MH propositions are more general than it appears. However, when transportation rates are a function of quantity shipped, we show that MH1 holds, but MH2 need not hold. This indicates that the presence of quantity discount in the transportation rates has a significant influence on the location decision of undifferentiated oligopoly.

Finally, it is of interest to point out that our analysis has generalized recent discussions on the problem of plant location and production decisions under oligopoly in the (a-3)

sense that the conventional results can be easily obtained from our model by assuming that transportation rate are independent of quantity shipped.

Appendix

In this appendix, we derive (10), (11) and (13). Dividing (7) by (8), we obtain

$$(w + ksu_L)/(r + mzu_K) = f_L/f_K$$
(a-1)

Hence, we can rewrite the first-order conditions as

$$Af_{K} - Bf_{L} = 0 \tag{a-2}$$

q - f(L, K) = 0

where $A \equiv w + ksu_L$ and $B = r + mzu_K$.

Totally differentiating (a-1) and (a-2), we obtain

$$\begin{bmatrix} AfKL - BfLL + kLsuLfK - fL \\ AfKK - BfLK + mKzuKfL - fK \end{bmatrix} \begin{bmatrix} dL \\ dK \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix} dq \quad (a-4)$$

where $k_L \equiv \partial k/\partial L \ m_K \equiv \partial m/\partial K$. Using the property of the homogenous production function of degree n, $f_{LL}L + f_{LK}K = (n-1)f_L$ and $f_{LK}L + f_{KK}K = (n-1)f_K$, and via Cramer's rule, we obtain

$$dL/dq = (1/D)(Af_{KK} - Bf_{LK} - m_K z u_K f_L) \neq L/nq$$
 (a-5)

$$dK/dq = (1/D)(Bf_{LL} - Af_{KL} - k_L su_L f_K) \neq K/nq$$
 (a-6)

where

$$D = (1/L)[nq(Af_{KK} - Bf_{LK}) - (f_{K}^{2}k_{L}su_{L} + f_{L}^{2}m_{k}zu_{K})L]$$

= (1/K)[nq(Bf_{LL} - Af_{KL}) - (f_{K}^{2}k_{L}su_{L} + f_{L}^{2}m_{k}zu_{K})K] (a-7)

If transportation rates are independent of quantity shipped, $k_L = 0$ and $m_K = 0$. Substituting these conditions into (a-5), (a-6) and (a-7), we obtain

$$dL/dq = L/nq$$
 and $dK/dq = K/nq$ (a-8)

Note that (a-5) and (a-6) are (10) and (11) and (a-8) is (13).

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