American Journal of Business, Economics and Management
2015; 3(3): 162-170
Published online June 10, 2015 (http://www.openscienceonline.com/journal/ajbem)

Neural Network Approach to Estimating Conditional Quantile Polynomial Distributed Lag (QPDL) Model with an Application to Rubber Price Returns

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To cite this article

Kwadwo Agyei Nyantakyi. Neural Network Approach to Estimating Conditional Quantile Polynomial Distributed Lag (QPDL) Model with an Application to Rubber Price Returns. *American Journal of Business, Economics and Management.* Vol. 3, No. 3, 2015, pp. 162-170.

Abstract

In this paper we consider the estimation of the conditional quantile polynomial distributed lag (QPDL) using neural network to investigate the influence of the conditioning variables on the location, scale and shape parameters of the QPDL model developed. This method avoids the need for a distributional assumption and applies conditional quantiles approach which allows the investigator to employ a range of conditional functions which exposes a variety of forms of conditional heterogeneity to give a more comprehensive picture of the effects of the independent variable on the dependent variable. The models fitted were adequate with very high R-square and low AIC values across the quantiles. We observe the effects of the quantiles on the dependent variables through the various GAM-style plots. Also from the actual and the predicted plots we observed that there was no difference between them. The results suggest that neural network used in estimating the QPDL model offers a useful alternative for estimating the conditional density, as artificial neural networks have proven to produce good prediction results in regression problems.

Keywords

Backpropagation, Effects, Feedforward, Hidden Neurons, Polynomial Transformation

1. Introduction

Artificial neural networks represent an excellent tool that have been used to develop a wide range of real-world applications, especially when traditional methods of solving problems fail (Scott, 1993)[13]. Artificial neural networks exhibit advantages such as ideal learning ability from data, classification capabilities and generalization for situations not contained in training data set, (Thrun, 1994)[16], computationally fastness once trained due to parallel processing, noise tolerance (Towell, 1993)[17]. This method avoids the need for a distributional assumption. Statistical aspects of artificial neural networks, such as approximation and convergence properties, have been discussed, and compared with properties of more "classical" methods (Barron and Barron, 1988; Geman et al., 1992; Ripley, 1993)[2,7,14]. Artificial neural networks have proven to produce good prediction results in regression problems Ripley, (1996)[15].

At the heart of statistics lies regression. Ordinary least squares regression estimates the mean response as a function of the regressors or predictors, and least absolute deviation regression estimates the conditional median function, which has been shown to be more robust to outliers (Koenker, 2005)[11], while ordinary least squares regression is not effective in capturing the extreme values or the adverse losses evident in return distributions, these are captured by quantile regressions. Basset and Koenker (1982)[10], introduced quantile regression to estimate the conditional quantile function of the response, by using the idea of generalized least absolute deviation regression. As a result, they provide much more information about the conditional distribution of a response variable

In this paper we used artificial neural networks with simple architecture (one or two hidden layers) because they can approximate any function and its derivatives with any desired accuracy (Cybenko, 1989; Hornik et al., 1990;

Hornik et al., 1993)[4,8,9].

The objectives of this study are, to estimate the Conditional Quantile Polynomial distributed lags using neural network, to investigate the influence of the conditioning variables on the location, scale and shape parameters of the QPDL model, and to study the effects of rubber production on its price returns at different quantiles.

2. Materials and Methods

2.1. Data Source

Secondary annual data was collected from FAOSTAT [5], food balance sheet, price statistics, available with the Department of Census and Statistic Sri Lanka, and the World Bank (pink sheet)[18]. These data comprises of the production, imports, exports and prices of rubber. The rubber data ranges from 1961-2011.

2.2. Statistical Software

The R software, with the package quantile neural network (Qrnn) was used in fitting the conditional quantile polynomial distributed lag models.

2.3. Methodology

Given the autoregressive distributed lag

$$Y_t = \alpha + \beta_0 X_t + \beta_1 X_{t-1} + \beta_2 X_{t-2} + \beta_3 X_{t-3} + --- + \beta_n X_{t-k} + \varepsilon_t \quad (2.1)$$

With Y_t dependent variable and X_t independent variable and $\{\varepsilon_t, 1 \le t \le k\}$ are independent identically distributed random errors.

$$X_{t-1} = X_{t-2}, - - -, X_{t-k},$$

$F_{t-1} = \sigma\{X_u, u \le t-1\}$ the $\sigma - algebra$ generated by the observations up to time $t-1$.

The polynomial distributed lag by Almon, (1965)[1] can be written as

$$Y_t = \varphi + \sum_{i=0}^{k} \beta_i X_{t-i} + \varepsilon_t \quad (2.2)$$

With $k\ number\ of\ lags$, and the $\beta_i's$ can be approximated by suitable polynomials. That is

$$\beta_i = a_0 + a_1 i + a_2 i^2 + a_3 i^3 + --- + a_n i^n$$

Thus for the n[th] degree polynomial with $k\ number\ of\ lags$ we have

$$Y_t = \varphi + \sum_{i=0}^{k} (a_0 + a_1 i + a_2 i^2 + a_3 i^3 + --- + a_n i^n)\, X_{t-i} + \varepsilon_t$$

$$Y_t = \varphi + \sum_{i=0}^{k} a_0 X_{t-i} + \sum_{i=0}^{k} a_1 (i) X_{t-i} + \sum_{i=0}^{k} a_2 (i^2) X_{t-i} + \sum_{i=0}^{k} a_3 (i^3) X_{t-i} + - - - + \sum_{i=0}^{k} a_n (i^n) X_{t-i} + \varepsilon_t \quad (2.3)$$

as defined by Koenker and Xiao (2002)[12] and Galvao Jr, , Montes-Rojas, & Park, (2009)[6], Bassett and Koenker,(1982)[3]. Given a uniformly standard random variable identically independently distributed sequence of random variables $\{\psi_t\}$, we define the new model as conditional quantile polynomial distribute lag (QPDL) for the n[th] degree polynomial as given below:

2.4. Conditional Quantile Polynomial Distributed Lag Model (QPDL)

$$Y_t = \varphi(\psi_t) + a_0 \psi_t \sum_{i=0}^{k} X_{t-i} + a_1 \psi_t \sum_{i=0}^{k} (i) X_{t-i} + a_2 \psi_t \sum_{i=0}^{k} (i^2) X_{t-i} + a_3 \psi_t \sum_{i=0}^{k} (i^3) X_{t-i} + - - - + a_n \psi_t \sum_{i=0}^{k} (i^n) X_{t-i} + \varepsilon_t \quad (2.4)$$

$where$ $\{\varepsilon_t, 1 \le t \le n\}$ are independent identically distributed random errors.

Then the QPDL can be simplified as:

$$Y_t = \varphi(\psi_t) + a_0(\psi_t) Z_{0t} + a_1(\psi_t) Z_{1t} + a_2(\psi_t) Z_{2t} + a_3(\psi_t) Z_{3t} + -- + a_n(\psi_t) Z_{nt} + \varepsilon_t \quad (2.5)$$

The conditional $\alpha - quantile$ function (QPDL) can be written as

$$Q_{y_t}(\alpha | Z_{0t}, - - -, Z_{nt}) = \varphi(\alpha) + a_0(\alpha) Z_{0t} + a_1(\alpha) Z_{1t} + a_2(\alpha) Z_{2t} + a_3(\alpha) Z_{3t} + - - - + a_n(\alpha) Z_{nt} + Q_{y_t}(\varepsilon_t). \quad (2.6)$$

Where $Q_{y_t}(\varepsilon_t) = u_t$.

For a QPDL (2) we have

$$Y_t = \varphi(\alpha) + a_0(\alpha) Z_{0t} + a_1(\alpha) Z_{1t} + a_2(\alpha) Z_{2t} + \varepsilon_t. \quad (2.7)$$

2.5. Neural Network

Let $\theta = (V_0, - - -, V_H, \alpha_1, - - -, \alpha_H, \beta_H)$ with $\alpha_j = (\alpha_{j1}, - - -, \alpha_{jp})$ then

$$f(Y_t, \theta) = V_0 + \sum_{h=1}^{H} V_h \varphi(< \alpha_h, Y_t > + \beta_h) \quad (2.8)$$

Denote a one layer feed forward neural network with H hidden neurons. Assuming φ to be twice continuous differentiable and belongs to the class of sigmoid functions satisfying

$$\lim_{x \to -\infty} \varphi(y) = 0, \lim_{x \to \infty} \varphi(y) = 1 \text{ and } \varphi(y) + \varphi(-y) = 1.$$

Example is the logistic function $\varphi(y) = (1 + e^{-y})^{-1}$

For the 2[nd] degree polynomial conditional quantile model given by (2.7)

$$Y_t = \varphi(\alpha) + a_0(\alpha)Z_{0t} + a_1(\alpha)Z_{1t} + a_2(\alpha)Z_{2t} + \varepsilon_t$$

where

$Z_{it} = (Z_{0t}, - - -, Z_{nt})$, $\theta_0$is a fixed but unknown, ε_t is independent of

$q_{t-1} = \sigma\{X_u, u \leq t-1\}$, the sequence generated by the observations up to time $t-1$.

Furthermore $\{\varepsilon_t, 1 \leq t \leq n\}$ are independent identically distributed random errors.

To estimate the conditional function q(Y_t) given the input

$Z_{it} = (Z_{0t}, - - -, Z_{nt})^T \in R^d$, using neural network with one hidden layer consisting of $H \geq 1$ neurons defined by the feedforward function

$f(Y_t) = f_H(Y_t, \theta)$ of the following form

$$f(Y_t, \theta) = V_{0j} + \sum_{h=1}^{H} V_h \varphi({Z_{it}}^T \omega_h, +\omega_{h0}) \qquad (2.9)$$

where $\omega_h = (\omega_{h1}, - - -, \omega_{hd})^T$. The activation function φ is fixed in advance with network weights, $V_0, - - -, V_H, \omega_{h1}, h = 1, - - -, H, i = 0, - - -, d$, which we combine to a M(H)-dimensional parameter vector θ with $M(H) = 1 + H + H(1+d)$, may be chosen appropriately. We denote the class of such neural network output functions by

$$0 = \{f_H(y;\theta); \theta \in R^{M(H)}, H \geq 1\}.$$

We consider a bipolar sigmoid function with range (-1,1) given by

$\varphi(y) = \frac{2}{1+\exp(-\sigma y)} - 1$ and $\varphi'(y) = \frac{\sigma}{2}[1+f(y)][1-f(y)]$ where σ is the steepness parameter. Then for the $Q_{y_t}(\alpha|Z_{it}) = {Z_{it}}^T \beta(\alpha)$ as defined before for QPDL(2), (2.7)

$$Y_t = \varphi(\alpha) + a_0(\alpha)Z_{0t} + a_1(\alpha)Z_{1t} + a_2(\alpha)Z_{2t} + \varepsilon_t$$

Using the activation function

$\varphi(y_t) = \frac{2}{1+\exp(-\sigma y_t)} - 1$ and $\varphi'(y_t) = \frac{\sigma}{2}[1+q(Z_{it})][1-q(Z_{it})]$ we can estimate the neural network by using the following algorithm and the input feed forward

$(Y_t, t = 1, - - -, n)$, and output target vector $(t = t_1, - - -, t_k, - - -, t_n)$, then for Z_j hidden unit z_j the net input to Z_j denoted by $z_{inj}; J = 1, - - -, p$

$z_{inj} = v_{0j} + \sum_{i=1}^{n} x_i v_{ij}$ where v_{0j} is the bias. The output signal (activation) of Z_j is denoted by z_j, $z_j = f(z_{inj})$.

Also for the Y_k for the output unit K: the net input to Y_k denoted by

y_{ink} , $k = (1, - - -, m)$, then

$y_{ink} = \omega_{0k} + \sum_{j=1}^{p} z_j \omega_{jk}$ with an output (activation) signal of Y_k given by

$y_k = f(y_{ink})$. This activation function was chosen because for a back propagation net, the activation function should be continuous, differentiable and monotonically non-decreasing and also easy to compute.

2.6. The Backpropagation Error

Each output unit $(Y_k, k = 1, - - -, m)$ the error δ_k is calculated by,

$\delta_k = (t_k - y_k)f'(y_{ink})$ and the weight correction term ω_{jk} is updated later, where

$\Delta\omega_{jk} = \alpha\delta_k z_j$ And the bias correction term is $\Delta\omega_{0k} = \alpha\delta_k$ therefore for the sum of the errors for the hidden units $(Z_j, j = 1, - - -, p)$ $\delta_{inj} = \sum_{k=1}^{m} \delta_k \omega_{jk}$ and multiplying by the derivative of the activation function we have , $\delta_j = \delta_{inj} f'(z_{inj})$ and the and the correction term weights are calculated by $\Delta v_{ij} = \alpha\delta_j x_i$ which update as $\Delta v_{ij} = \alpha\delta_j$

3. Results and Discussion

Neural network parameter estimates for the Conditional Quantile polynomial distributed lag (QPDL) with (3-lags) for rubber production, with Proz0 the first polynomial transformation for production, Proz1 the second polynomial transformation and Proz2 the third polynomial transformation, the standard errors and confidence intervals are given in Table 1, 2, 3 and 4. We present the table for tau=0.25, 0.50, 0.75 and 0.95.

Table 1. *Conditional quantile polynomial distributed lag (QPDL) .25 Quantile Neural network parameter estimates, standard errors and confidence intervals for production and price of rubber.*

Quantile	Price	Coef	Std. Err.	P>|t|	[95% Conf. Interval]
0.25	Proz0	16.37	7.68	0.000	1.32, 31.35
	Proz1	-14.31	7.79	0.000	-29.59, -0.94
	Proz2	-1.32	1.16	0.010	-2.48, -0.156
	Constant	-1.41	1.20	0.005	-2.61, 0.21

R-square=0.52, AIC=263.40, RMSE=501.38, P>F=0.000

The resulting estimated equation from Table 1 is:

$$\hat{Y}_t = -1.407 \ + 16.37Z_{0t} - 14.31Z_{1t} - 1.32Z_{2t} \qquad (3.1)$$

From Table 1, we observed that all the parameter estimates are significant at the 25th percentile. Also the R-square value is 52%, that is, the dependent variable can be explained by the independent variables with AIC=263.4. The resulting transformed equation is given below:

Estimating the coefficients of the original variables, we have

$$\hat{\beta}_0 = \hat{a}_0 = 16.373477$$

$$\hat{\beta}_1 = \hat{a}_0 + \hat{a}_1 + \hat{a}_2$$
$$= 16.373477 - 14.313269 - 1.317999$$
$$= 0.742209$$

$$\hat{\beta}_2 = \hat{a}_0 + 2\hat{a}_1 + 4\hat{a}_2$$
$$= 16.373477 - (2 * 14.313269)$$
$$- (4 * 1.317999) = -17.525057$$

$$\hat{\beta}_3 = \hat{a}_0 + 3\hat{a}_1 + 9\hat{a}_2$$
$$= 16.373477 - (3 * 14.313269)$$
$$- (9 * 1.317999) = -38.428321$$

Therefore we have the QPDL estimated model as:

$$\hat{Y}_t = -1.407 \; + 16.37X_0 + 0.74X_{t-1} - 17.53X_{t-2} - 38.43X_{t-3} \quad (3.2)$$

Table 2. *Conditional quantile polynomial distributed lag (QPDL) .50 Quantile Neural network parameter estimates, standard errors and confidence intervals for production and price of rubber.*

Quantile	Price	Coef	Std. Err.	P>|t|	[95% Conf. Interval]
0.50	Proz0	4.67	1.444	0.0002	1.84, 7.50
	Proz1	-3.81	2.430	0.0000	-7.43, -0.19
	Proz2	0.09	0..333	0.0000	-0.565, 0.744
	Constant	-0.71	0.644	0.0000	-1.97, 0.55

R-square=0.64, AIC=257.54, RMSE=435.63, P>F=0.000

The resulting estimated equation from Table 2 is:

$$\hat{Y}_t = -0.715 \; + 4.667Z_{0t} - 3.815Z_{1t} + 0.093Z_{2t} \quad (3.3)$$

From Table 2, we observed that all the parameter estimates are significant at the 50th percentile. Also the R-square value is high, that is 64% of the dependent variable can be explained by the independent variables with AIC=257.54. The resulting transformed equation is given below:

Estimating the coefficients of the original variables, we have

$$\hat{\beta}_0 = \hat{a}_0 = 4.666599$$

$$\hat{\beta}_1 = \hat{a}_0 + \hat{a}_1 + \hat{a}_2 = 4.666599 - 3.814709 + 0.092598 = 0.944488$$

$$\hat{\beta}_2 = \hat{a}_0 + 2\hat{a}_1 + 4\hat{a}_2 = 4.666599 - (2 * 3.814709) + (4 * 0.092598) = -2.592427$$

$$\hat{\beta}_3 = \hat{a}_0 + 3\hat{a}_1 + 9\hat{a}_2 = 4.666599 - (3 * 3.814709) + (9 * 0.092598) = -5.944146$$

Therefore we have the QPDL estimated model as:

$$\hat{Y}_t = -0.715 + 4.667X_0 + 0.944X_{t-1} - 2.592X_{t-2} - 5.944X_{t-3} \quad (3.4)$$

Table 3. *Conditional quantile polynomial distributed lag (QPDL) .75 Quantile Neural network parameter estimates, standard errors and confidence intervals for production and price of rubber.*

Quantile	Price	Coef	Std. Err.	P>|t|	[95% Conf. Interval]
0.75	Proz0	-15.26	5.8	0.000	-26.62, -3.90
	Proz1	11.04	3.92	0.021	3.32, 18.72
	Proz2	0.06	0.35	0.003	-0.62, 0.74
	Constant	3.53	1.00	0.000	1.57, 5.49

R-square=0.78, AIC=242.39, RMSE=302.87, P>F=0.0002

The resulting estimated equation from Table 3 is:

$$\hat{Y}_t = 3.529 \; - 15.261Z_{0t} + 11.037Z_{1t} + 0.058Z_{2t} \quad (3.5)$$

The 75th percentile from Table 3, all the parameter estimates for the lags and the constant are significant with R-square of 78% and AIC=242.39 and RMSE= 302.87.

Estimating the coefficients of the original variables, we have

$$\hat{\beta}_0 = \hat{a}_0 = -15.260809$$

$$\hat{\beta}_1 = \hat{a}_0 + \hat{a}_1 + \hat{a}_2 = -15.260809 + 11.036829 + 0.057543 = -4.166437$$

$$\hat{\beta}_2 = \hat{a}_0 + 2\hat{a}_1 + 4\hat{a}_2 = -15.260809 + (2 * 11.036829) + (4 * 0.057543) = 7.043021$$

$$\hat{\beta}_3 = \hat{a}_0 + 3\hat{a}_1 + 9\hat{a}_2 = -15.260809 + (3 * 11.036829) + (9 * 0.057543) = 18.367565$$

Therefore we have the QPDL estimated model as:

$$\hat{Y}_t = 3.529 - 15.261X_0 - 4.166X_{t-1} + 7.043X_{t-2} + 18.368X_{t-3} \quad (3.6)$$

Table 4. *Conditional quantile polynomial distributed lag (QPDL) .95 Quantile Neural network parameter estimates, standard errors and confidence intervals for production and price of rubber.*

Quantile	Price	Coef	Std. Err.	P>|t|	[95% Conf. Interval]
0.95	Proz0	-7.73	3.36	0.0000	-14.33, -1.13
	Proz1	4.96	1.55	0.0000	1.91, 8.01
	Proz2	0.57	0.418	0.0031	0.461, 0.86
	Constant	2.33	0.536	0.0022	1.28, 3.38

R-square=0.85, AIC=298.35, RMSE=1159.37, P>F=0.000

The resulting estimated equation from Table 4 is:

$$\hat{Y}_t = 2.329 \; - 7.727Z_{0t} + 4.956Z_{1t} + 0.574Z_{2t} \quad (3.7)$$

From Table 4, we observed that all the parameter estimates are significant at the 95th percentile. Also the R-square value is high, that is about 85% of the dependent variable can be explained by the independent variables with AIC=298.35. The resulting transformed equation is given below:

Estimating the coefficients of the original variables, we have

$$\hat{\beta}_0 = \hat{a}_0 = -7.727470$$

$$\hat{\beta}_1 = \hat{a}_0 + \hat{a}_1 + \hat{a}_2 = -7.727470 + 4.956433 + 0.573748 = -2.197289$$

$$\hat{\beta}_2 = \hat{a}_0 + 2\hat{a}_1 + 4\hat{a}_2 = -7.727470 + (2 * 4.956433) - (4 * 0.573748) = -0.109596$$

$$\hat{\beta}_3 = \hat{a}_0 + 3\hat{a}_1 + 9\hat{a}_2 = -7.727470 + (3 * 4.956433) - (9 * 0.573748) = 1.978097$$

Therefore we have the QPDL estimated model as:

$$\hat{Y}_t = 2.329 - 7.727X_0 - 2.197X_{t-1} - 0.110X_{t-2} + 1.978X_{t-3} \quad (3.8)$$

3.1. Graphical Interpretation of the Effects

We present the graphical effects of only tau=0.25 on x1=proz0, x2=proz1 and x3=proz2, and the plots of actual and the predicted values against x1=proz0, x2=proz1 and x3=proz2.

GAM-style effect plots provide a graphical means of interpreting fitted predictor/predictor relationships. From Koenker et al. (2002)[12]: The effect of the ith input variable at a particular input point is the change in function resulting from changing X1 to x1 from b1 (the baseline value) while keeping the other inputs constant. The effects are plotted as short line segments, and the slope of the segment is given by the partial derivative. Functions without interactions appear as possibly broken straight lines (linear functions) or curves (nonlinear functions). Interactions are present when the effect of a variable depends on the values of other variables and they show up as vertical spread at a particular horizontal location, that is, a vertical scattering of segments.

Figure 1, 2 and 3 shows the plots of the effects of tau=0.25 on x1=proz0, x2=proz1 and x3=proz2

From figure 1 we observe that, x1 strongly influence the function value since it has a large total vertical range of effects. Hence there is a large effect of the independent variable on the dependent variable with no interaction between x1 and the other independent variables x2 and x3. Thus the effect of x1 does not depend on the values of other variables as shown by the broken vertical line.

From figure 2, we observe a similar large effect of x2 on the dependent variable. There is no interaction between x2, x1 and x3as shown by the vertical line.

From figure 3 we observe that, there is a large effect of the independent variable on the dependent variable as shown by the vertical line with no interaction between x3 and the other independent variables x2 and x1.

3.2. Graphical Interpretation of the Plots of Actual and the Predicted Values

Figure 4, 5 and 6 shows the plots of actual values(black) and predicted values (red) against variables x1=proz0, x2=proz1 and x3=proz2.we present only the results for tau=0.25

From figure 4, we observe the plot of the actual (black) values and the predicted (red) values against the x1=proz0, and there is no significant difference between the actual and the predicted values since they show similar clustering pattern.

From figure 5, we observe similar pattern between the actual and the predicted values with very few outliers of the actual values from the predicted values. Thus, there is no significant difference between the actual and the predicted values, as they have similar cluster pattern.

From figure 6, we observe similar pattern between the actual and the predicted values. There is no significant difference between the actual and the predicted values, as they have similar cluster pattern.

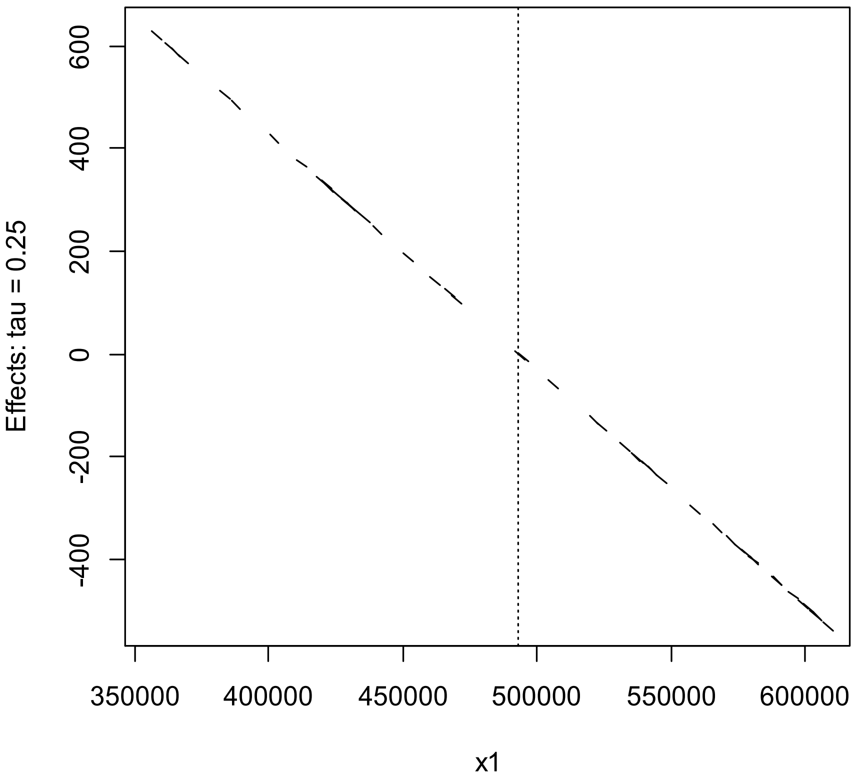

Figure 1. *Plot of effects of tau=0.25 on x1.*

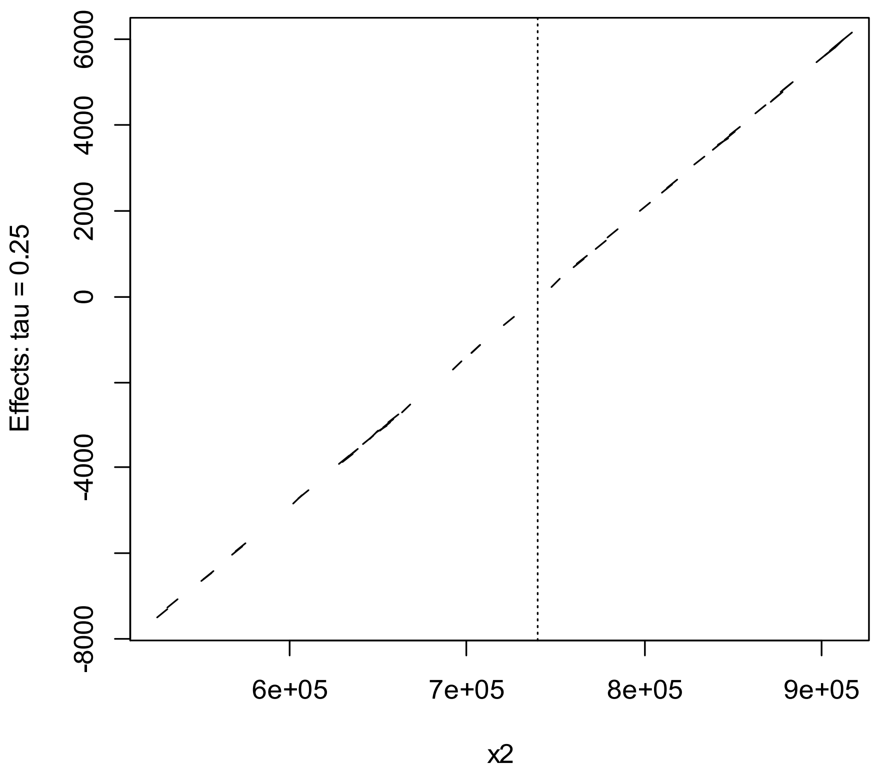

Figure 2. *Plot of effects of tau=0.25 on x2.*

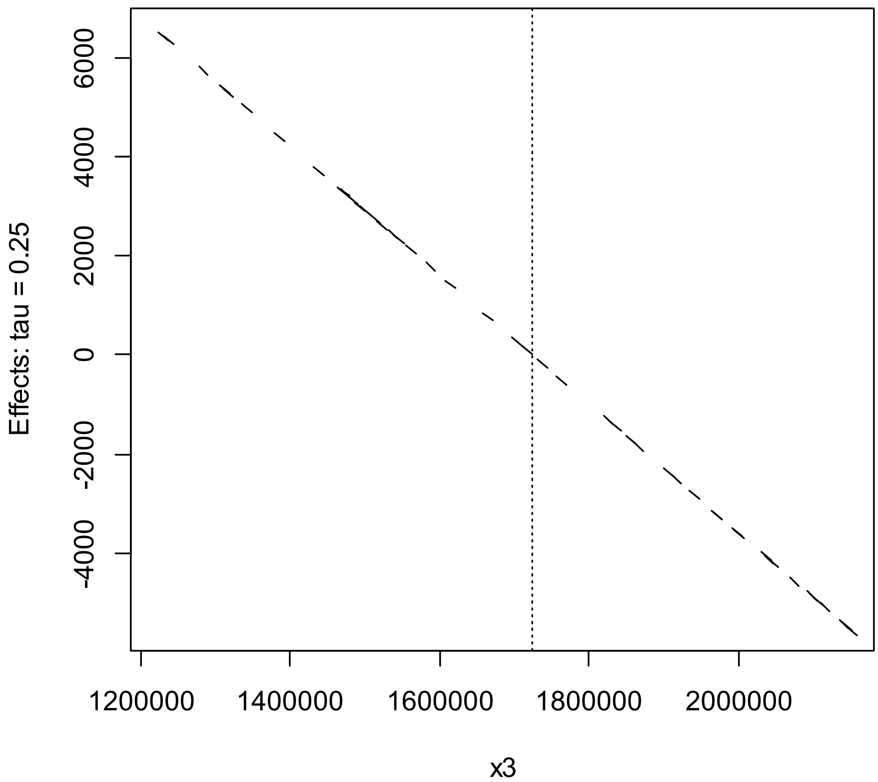

Figure 3. *Plot of effects of tau=0.25 on x3.*

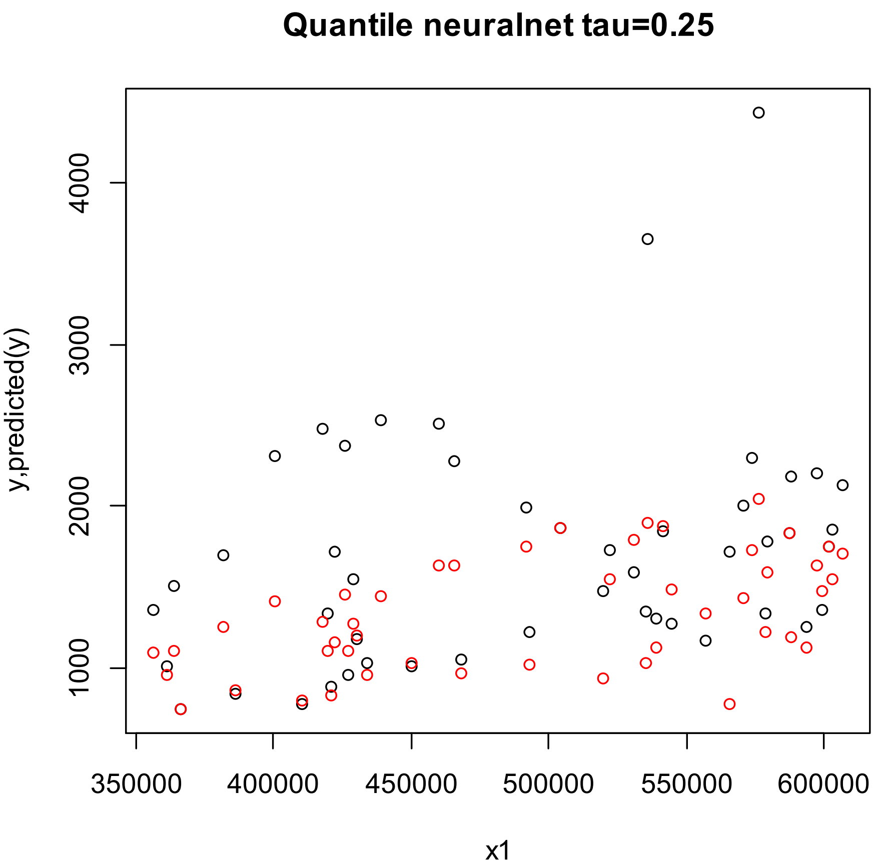

Figure 4. *Plot of (y, predicted(y)) against X1.*

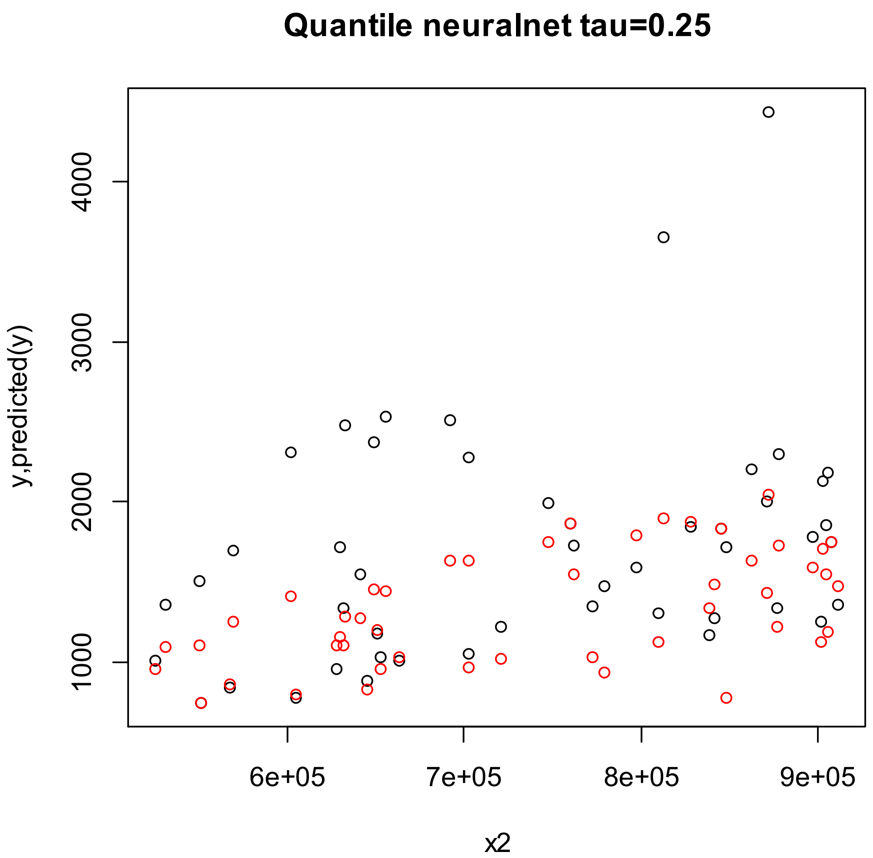

Figure 5. *Plot of (y, predicted(y)) against X2.*

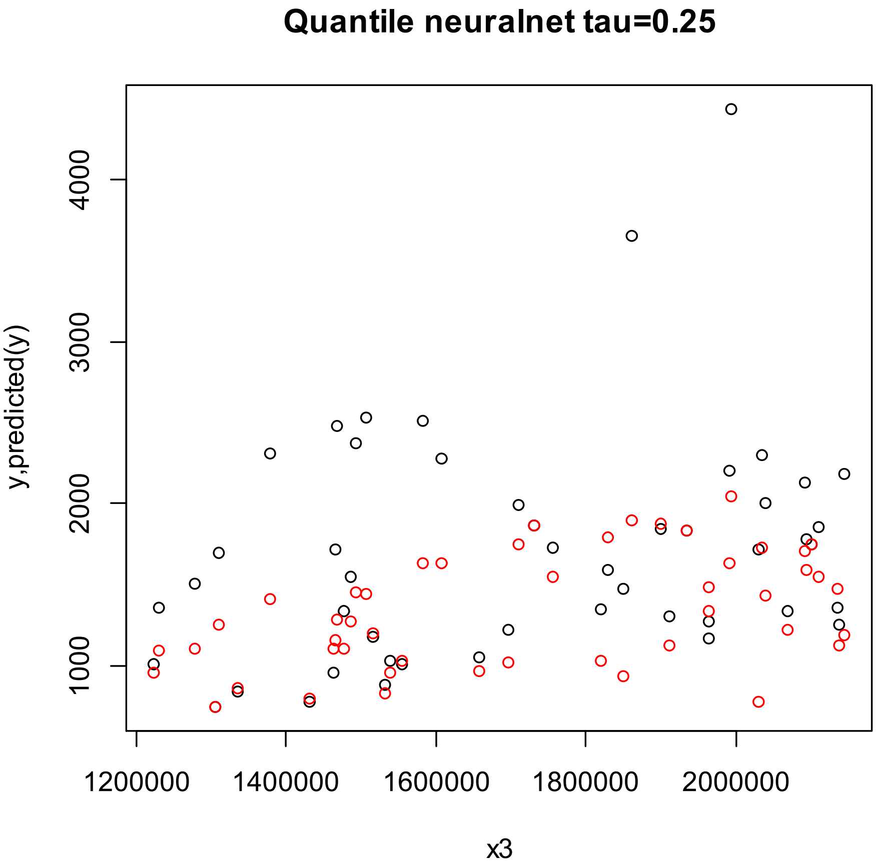

Figure 6. *Plot of (y, predicted(y)) against X3.*

4. Conclusions

We have presented a nonparametric approach to estimating the conditional polynomial distributed lag. Neural Network can learn to approximate any function just by using example data that is representative of the desired task. We observe that all the parameter estimates were significant. The models fitted were adequate with very high R-square and low AIC values across the quantiles. We observe the effects of the quantiles on the dependent variables through the GAM-style effect plots, which provide a graphical means of interpreting fitted predictor/predictor relationships. Also from the actual and the predicted plots we observed that there was no significant difference between them. We also observed that the quantile regression provides a more complete picture of the conditional distribution and can be used for modelling. The results suggest that neural network used in estimating the QPDL model offers a useful alternative for estimating the conditional density, as use of artificial neural networks in this study have also proven to produce good prediction results in regression problems.

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