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Ultrasonic Study of Nonlinear Internal Friction and Creep in Rocks Under Uniaxial Stress

Valery Tsaplev[1, 2, *], R. Konovalov[1], R. Ivanyuk[1], Victor Zverevich[3], Grigory Zhukov[3]

[1]Dep. of Electroacoustics and Ultrasonics Engineering, Saint-Petersburg State Electrotechnical University (LETI), Saint-Petersburg, Russia
[2]Dep. of Physics, North-West Open Technical University, Saint-Petersburg, Russia
[3]Dep. of Mine Surveying, National Mineral Resourses University ("Gorny"), Saint-Petersburg, Russia

Email address

valery@convergences-fr.ru (V. Tsaplev), vmcaplev@etu.ru (V. Tsaplev), rskonovalov.eut@gmail.com (R. Konovalov), rivanyuk@phosagro.ru (R. Ivanyuk), kmd@spmi.ru (V. Zverevich), zhukov_gp@spmi.ru (G. Zhukov)

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Abstract

The influence of the uniaxial static stress on the internal friction of different polycrystalline rocks was studied using the ultrasonic method of loaded complex oscillator. Different rocks with different coefficients of internal friction were studied – granite, dolomite, granulite, gabbro, hibinite, quartz, quartzite. Strong dependence of the internal friction coefficient from the uniaxial stress was found. These changes are accompanied by essential creep. Possible reasons for these phenomena are discussed.

Keywords

Polycrystalline Rocks, Microporosity, Microcracks, Nonlinear Elasticity, Creep, Ultrasonic Control, Internal Friction, Uniaxial Stress, Ultrasonic Method, Loaded Complex Oscillator

1. Introduction

The research of natural conditions of mining works is of primary value for the designing, construction and maintenance of underground structures. The strength of ore and enclosing strata, the stability of rock massif affect essentially on the performance of the whole system of the underground structures [1], [2]. The monitoring of natural conditions is impossible without knowledge of physical and mechanical properties of ore and enclosing strata. This paper presents in a nut shell some results of the study of one of the main properties of rocks – the dependence of their elastic modulus on essential constant stress.

The stress dependences of the elastic moduli are very important mechanical properties of rocks [3], [4]. They also characterize their physical properties. The internal friction [5] seems to be an associated factor, which is not the main quality to estimate the strength of underground constructions, nevertheless is a very important characteristic of the state of ore and enclosing strata. And vice versa, when studying physical properties of rocks the information about the internal friction is of the primary importance, it may be even more important than the information about elastic moduli, because it strongly depends on the physical interactions inside the material.

That is why the study of prestressed material is a valuable method in the materials science [6] and gives very important data about the physical nature of material and about the processes that go on inside it [7].

Many scientists [8], [9], [10], [11] studied the effect of load on the properties of rocks, and it was found, that the elastic moduli strongly depend on the static stress, both uniaxial, biaxial [12] and uniform compression. Rocks manifest clearly marked elastic nonlinearity.

It was also found, that rocks manifest elastic creep [13], though the law, that describes changes of their elastic and unelastic properties in time, is still not properly established despite numerous researchers of this effect [14], [15], [16], [17].

In the steady state, after the process of creep is finished (the time of transient period may be either short or very long, depending on the material, temperature and stress) the elastic parameters are stabilized.

The time of extinction of intragenic changes and steady parameters (if the load did not exceed the breaking point) should be measured as accurate as possible.

The measurements of elastic properties and the internal friction under the influence of the constant uniaxial stress were possible by means of the specially designed method [18]. One must mind, that the compressibility of rocks strongly depends not only on their polycrystalline structure [6], but also their porosity and microfracturing [19], [20], [21]. These defects affect the elastic moduli of higher order and the internal friction. If the porosity is open, then its value is close to the value of the veritable porosity. The question about the identification of the porosity was discussed by a number of scientists [5], [6], [23], [24] and others. If the pores are closed, then the difference between their values may be sufficient.

The real methods of the evaluation of porosity do not give the possibility of the differentiation between the porosity and the. It is only possible to estimate the empty to solid phase ratio of the material. The microfracture volume can be estimated only by means of the petrographic analysis.

Microfractures have more effect on the elasticity than the porosity. They also have more effect on the dependence of the elastic properties and the internal friction from the external and internal stresses, on such important parameters as water absorption, water permeability, strength, abradability, frost resistance, preservation and maintainability.

For microcracks one cannot say, that the changes of the boundaries are small under loading that is why the problem is nonlinear from the very beginning [25].

The growth of a microcrack is caused by imperfections or damages, which already exist inside the material. The formation of a microcrack activates energy release. The more is the microcrack, the more is the energy output [26].

These processes are well described in a number of scientific publications. Very detailed data on dependences of elastic moduli for different rocks were given in [8], [25]. These studies were also provided by V. Merkulova and V. Tsaplev [27], [28].

In [8] one can find the results of study of the Young's modulus defect (i. e. its relative change) under the uniaxial compression for granite from the Quincy deposit, Massachusetts. Under the uniaxial compression up to 8 MPa the Young's modulus defect reaches 140 %, with simultaneous manifest of high nonlinearity and creep. Such essential Young's modulus defect was observed in materials with considerable porosity and/or microfracturing. The effect can be explained by closing of the microcracks under stress. This, in turn, causes the decrease of internal friction, i.e. the losses of elastic energy at the microcrack boundaries [27], [28].

2. Usage of the Loaded Complex Oscillator

All the experimental data which was cited in the above mentioned papers was obtained by static measurements, or by dynamic methods, but under high damping impact of the compressing machine. The accuracy of data obtained under these conditions is very low, and using static methods it is only possible to evaluate the principal trend to change the parameter. We have used much more accurate ultrasonic resonance method of evaluation the Young's modulus defect (i.e. its relative change) and relative change of the internal friction under the action of constant static uniaxial stress to control the elastic nonlinearity caused by the presence of microcracks. Sixth different facing rock materials were chosen as specimens. All these rocks were taken from the Lovchorr deposit, Kirovsk, Murmansk region. These rocks were: Khibinite, Gabbro, Garnet-Amphibole Gneiss (Granulite), Granite, Dolomite and Quartzite

Ultrasonic resonance method of measuring the velocity of elastic waves and the internal friction under the action of constant static uniaxial stress was designed by one of the authors of this paper and formerly was used to study piezoceramic materials [14], [15]. This method is a modification of multicomponent loaded complex oscillator having low level of losses in support system.

The apparatus uses the resonance frequency method within the frequency range 20 - 250 kHz and the range of uniaxial stresses $0 \div 1.5\times10^8$ Pa. It is possible to measure the elastic modulus and the internal friction under the simultaneous influences of several factors - constant uniaxial stress, temperature, electric and/or magnetic static or alternating fields, large mechanical alternating stresses, etc.

One can also study the time-dependent behavior of materials (creep or aging) when one, or all of these factors are maintained constant, but these effects are also out of the scope of this paper.

In the frequency range mentioned, the complex oscillator method must be used and the main difficulty is to design a sample mounting system that permits uniaxial compression up to very high stresses, leaving at the same time the sample "acoustically free". That means that the energy losses through the mounting system must be small enough so they can be neglected compared to the internal losses inside the whole oscillator.

Figure 1 illustrates the method. Sample 1 is a tuned half-wavelength bar, the cross-section of which is small enough compared to its length. If it is necessary to make measurements at higher frequencies, it is possible to use the third or the fifth oscillation mode. In this case the length of the specimen bar must be equal to three or five half-wavelengths. The ends of the bar are ground parallel one to one another and strictly perpendicular to its axis.

The length of the sample is equal to one-half of the wavelength of the resonance frequency of the material of the bar. In fact, the diameter of the sample is 5 – 6 mm and its length depends on the velocity of the sound and the resonance frequency of the sample.

The sample is located between two similar half-wavelength (or three- or five-half-wavelengths) steel bars (2). The ends of these bars are also strictly parallel to each other and perpendicular, with a very high degree of accuracy, to the axis. The length of each steel bar is also one half of the length of the wave and their ends are also strictly parallel to each other and perpendicular to the axis of the bar.

The whole oscillator is compressed by the force F through two support flanges that are located at the nodal planes of tuned half-wavelength steel bars. Each flange is supported by three small steel hard spheres to provide "three-point" support in order to prevent or minimize losses of the oscillator energy through the supports.

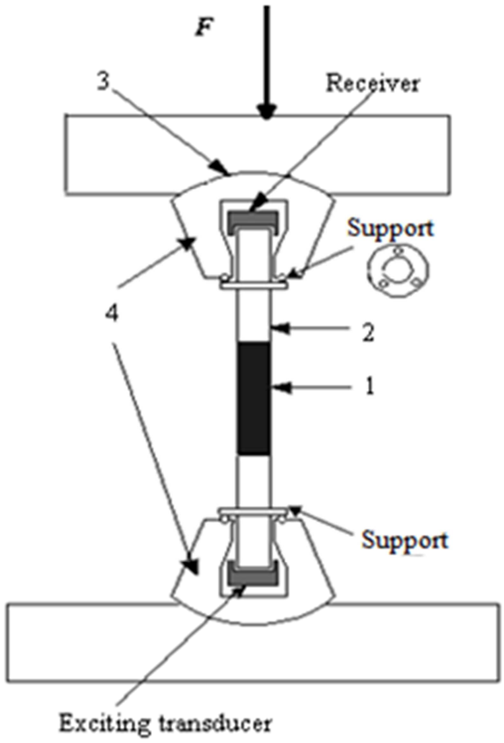

***Fig. 1.** Experimental setup.*

It is not necessary to say that the whole oscillator must be thoroughly tuned so that its resonance frequency is equal to that of each component with an accuracy of up to 1 – 2%. The ends of each component should be perpendicular to its axis and each axis should coincide with that of the whole oscillator. But still, as this coincidence cannot be absolute, the oscillator is compressed through two spherical joints (3) (one from each side) to take up possible clearances and non-parallelisms between the ends of the bars.

It is necessary to pay special attention to accuracy in assembling the whole oscillator, because the axial stability of the vibrator strongly depends on it.

The whole oscillator is excited by the electrodynamic transducer and its longitudinal oscillations are received by the transducer-receiver. Both the exciting transducer and the receiving one are located inside the steel supporting bodies (4) to provide the shielding from the unwanted signals.

The experimental setup provides measurements at the room temperature. It is possible also to measure the resonance frequency and the internal friction within necessary range of temperatures. In that case one must use a special heater or cooler to maintain the necessary temperature and it is necessary to insulate thermally the steel bars from the specimen, because the internal friction in steel increases significantly with rising of temperature. The Young's modulus of steel also changes significantly with the change of temperature. "Buffer" bars of fused quartz provide thermal insulation to prevent steel bars from heating and thus from increasing additional energy losses. The increase of energy losses in the bars of fused quartz when heating is small compared with losses in the specimen and these can be checked by preliminary frequency calibration.

The resonance frequency of the whole oscillator is measured by frequency meter. The output voltage from the transducer–receiver is amplified by the amplifier and recorded by a computer. The latter can control the oscillator with the help of the phase shifter and bandpass filter unit. The phase shifter is necessary to tune the whole circuit to the self-exitation mode. The signal that is received can be viewed by an oscilloscope. For high-amplitude measurements the set must have a pulse modulator and a high power amplifier.

The process of calculating the value of the Young's modulus and the value of the internal friction of the specimen from the experimental data of frequency and the internal friction of the whole oscillator is described in [29] and here we only present the experimental results.

The specimens for study were chosen from different horizons mining and with different level of microcracking.

The initial level of microcracking was measured in advance by means of the petrographic analysis. Before the loading the preliminary measurements of the elastic wave velocities and the internal friction for a number of control unloaded specimens were performed by using the three-component complex oscillator method [31]. The reference specimen was a fused silica rod.

The results of study of the dependences of the internal friction from the porosity factor and from the microcracking factor for some of mentioned above rocks were discussed in 18]. Some quartz specimens having zero porosity and very small internal friction losses were also studied in comparison. The results are presented in Table 1.

***Table 1.** Initial properties of unloaded rocks.*

Rock	Khibinite	Gabbro	Granulite	Granite	Dolomite	Quartzite	Quartz
$v\times10^3$, m/c	4,49	6,40	6,92	4,80	5,59	5,54	5,93
Q^{-1}	0,0088	0,0041	0,0074	0,0062	0,011	0,0025	< 0,0001
Q	116	243	135	161	91	400	>10000
$E\times10^{10}$, Pa	5,359	12,6	15,0	6,08	8,38	8,10	7,74
$\rho\times10^3$, kg/m^3	2,7	3,10	3,13	2,64	2,73	2,64	2,20
Porosity, %	0,70	0,46	0,62	0,56	0,88	0,35	0
Poisson's ratio	0,29	0,25	0,29	0,25	0,33	0,09	0,16

3. Experimental Results

The corresponding dependences were provided in [18]. As one can expect, the presence of pores and microcracks, i.e. the empty phase, affects considerably on the internal friction which increases as the relative volume of the empty phase grows. The defect (i.e. the relative change) of the elastic modulus also goes up. But one can hardly separate the influence of the porosity and of the microcracks on the internal friction and on the elastic modulus using only the ultrasonic measurements without the action of stress. A lot of factors affect on the sound velocity and on the internal friction, including effects of porosity, microcracking and intercrystalline interactions.

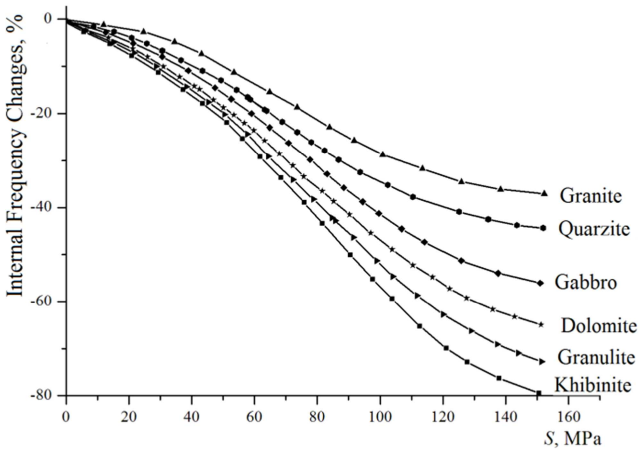

Fig. 2. Internal Friction Q^{-1} relative changes from stress.

Standard methods do not make it possible to separate these effects. The picnometer method and the method of hydrostatic weighing provide only the evaluation of the empty phase percentage with very low accuracy. Both pores and microcracks are the empty phase, but their effects on the elastic modulus and on the internal friction of rocks are different.

The porous media under low uniaxial stress first manifests linear elasticity. Microcracks arrive under the action of rather high stress and the stress dependences of the elastic modulus and the internal friction become nonlinear. The quasi-transversal microcracks tend to close under the action of longitudinal stress.

The threshold of destruction also increases.

On the contrary, the quasi-longitudinal microcracks tend to grow under the action of longitudinal stress, the Young's modulus decreases, the internal friction increases and the threshold of nonlinearity goes down.

The threshold of the beginning of destruction decreases.

Figure 2 shows the dependences of internal friction Q^{-1} relative changes from the uniaxial stress for different rocks. One can see that in general these curves correspond to the dependences of the internal fiction from the porosity which was studied in our recent work [18].

Khibinite, the most porous material, manifests the greatest stress dependence of the internal friction. On the contrary, less porous granulite, manifests the least stress dependence of the internal friction. This makes it possible to assume that the microcracking structure has more influence on the internal friction than the porosity, but this hypothesis needs an additional verification using alternative studies.

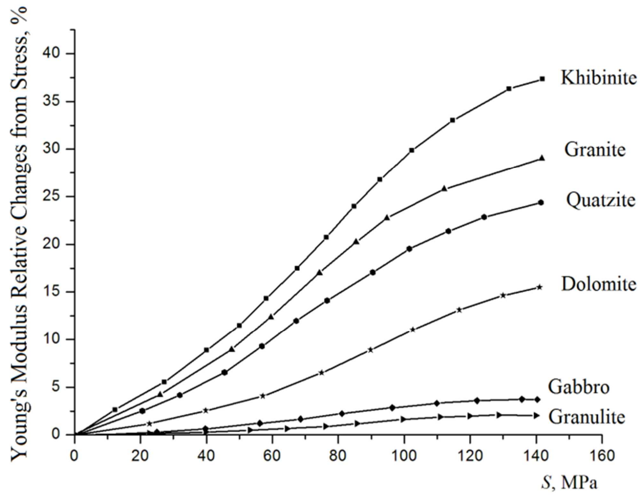

Fig. 3. Young's modulus relative changes from stress.

The relative changes of the dynamical Young's modulus (the "Young's modulus defect") under the action of longitudinal stress were measured using the same experimental setup. Figure 3 shows these dependences. The most porous khibinite manifests the greatest dependence, but the behavior of other rocks is different. For example granulite, which manifests the same dependence of the internal friction as the porous khibinire, demonstrates the minimal stress dependence of the Young's modulus.

All these dependences were obtained at the frequency 30 kHz, at the room temperature (300 K). All rocks manifested the long creep that is why the results were obtained after some time of load application, when all the time processes were finished.

4. The Study of Creep in Rocks

The first studies of elastic creep in rocks under the action of long-time constant stress were described by T. A. Read [32], [33]. It was found, that the elastic strain u follows the logarithmic law within the accuracy of measurements:

$$u = A \pm B \ln t + Ct\,,$$

where t – is time; A, B and C – constants evaluated experimentally.

Signs "plus" (+) and "minus" (–) are for two different types of processes in time: elastic and pseudo-viscous process, that take place simultaneously. The logarithmic law of creep is common for multyrelaxation systems.

Further studies of creep in rocks were provided by a great number of scientists (D.M. Cruden [34], J.C. Jaeger and

N.G.W. Cook [35], M.B. Dusseault and C.J. Fordham [36], etc.) and in general the proposed logarithmic law was confirmed, though some other models were also assumed. For example, L. Ma and J. Daemen [37] studied the creep of welded tuff and supposed three periods of the whole process.

During the first period quick changes take place. After the applying the axial stress, the axial strain increases with a high initial rate. The strain rate may be approximated by a power function of time

$$\varepsilon_1 = \varepsilon_0 + \alpha t^{\beta},$$

where ε_0 is the initial axial strain, α and β are material constants. These constants are subjects to be measured experimentally.

This first (transient) period is rather short and its time duration depends on the initial axial stress and on the favorably orientation of microcracks. This well corresponds to L.S. Costin and D.J. Holcomb [38], who suggested that rock loaded in compression initially responds elastically. When the applied stress reaches a sufficient magnitude, the stress intensity factor associated with the most favorably oriented microcracks reaches some critical value and these microcracks begin to grow. This means, that the duration of this transient period, the beginning of the second period, the final rate of changes during the second period and the beginning of the failure process depend on the most favorably orientation of microcracks. If they mostly are oriented perpendicularly to the axis of compression, they can close up and the process slows down. If, vice versa, the preferable orientation is along the axis of compression, the microcracks begin to grow up and the process goes faster. This was verified experimentally by the author [27] using the artificial “tempering” the porcelain specimens.

After the first transient period the second – steady state creep begins. This period may be much longer and may go on for a long time. L. Ma [37] proposed the following empirical law for this stress-induced strain change:

$$\varepsilon_2 = \dot{\varepsilon} \times t,$$

where $\dot{\varepsilon}$ is the steady-state creep rate. And the resulting strain measured in the steady-state creep is the sum of this stress-induced strain and the temperature-induced strain ε_2, which is proportional to the temperature change from the initial level [39]. Thereby one can observe the linear grow of the strain during this steady-state period the temperature being constant:

$$\varepsilon = \varepsilon_1 + \varepsilon_2 \sim t.$$

This formula does not match [32], [33] and to our data. We observed the linear dependence on the logarithm of time during the steady-state period for all studied rocks. Moreover, the strain curves described in [37] may also be fitted by the logarithmic law if we convert them to the logarithmic, not the linear scale.

At last, the third period – accelerating creep, which takes place at the final stress level and leads to failure of the specimen. Accelerating creep progresses very quickly, avalanche-like and this marks the onset of the specimen failure. This process was studied by L. Ma [40] and well corresponds to the results obtained by R.I. Kranz and C.H. Scholz [42] and recently by P. Baud [43].

If we put aside the problems of failure and, thereafter the third period, one can propose the integrated multirelaxation model of the creep process. It corresponds to a great number of possible physical mechanisms among which we can save out the effects of porosity and microcracks. Their effects on the elastic and compressive strength of rocks were studied in [43].

We choose the Young’s modulus defect as the factor much more informative for creep characterization instead of static strain. Moreover, it can be measured much more accurately.

We have observed, that after applying the stepwise constant load the internal friction first falls down [14] and the efficient elastic modulus (Young’s modulus) comes up not instantly, but with delay in time. The delay is not exponential in time, but logarithmic within some interval of time.

This means, that the relaxation process is not a single one, but contains a number of simple exponential processes and has a dense spectrum of relaxation time constants.

Relaxation processes are all types of processes that can take place in polycrystalline media having all types of macro- and microdefects.

Figure 4 shows the results of study of the time dependence of Young’s modulus of Khibinite under the action of stepwise applied constant stress of 100 MPa. The law of time behavior is logarithmic.

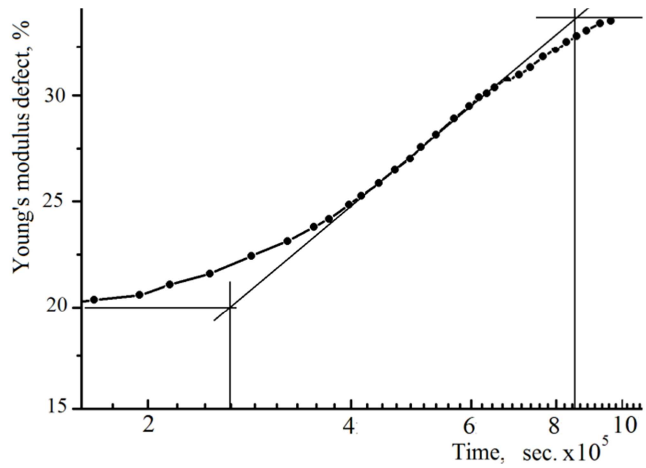

Fig. 4. *Young’s Modulus Time Change.*

The change of any parameter in time Δf in material having mechanical viscosity is possible to describe by n independent differential equations of relaxation type with independent coefficients:

$$\frac{d}{dt}(\Delta f_i) + \varphi_i \Delta f_i = \psi_i \text{ при } i = 1, 2, 3, \ldots, n, \quad (1)$$

$$\Delta f = \sum_{i=1}^{n} \Delta f_i ,$$

where φ_i and ψ_i – are functions of mechanical stress and amplitude, correspondingly, and $n \to \infty$.

If the stress changes in time as $\sigma(t) = \sigma_0 + \delta\sigma(t)$ and $\delta\sigma(t)$ is small enough, then changes of Δf_i are also small (δf_i) around $\Delta f_i^0 = \Delta f_i(\sigma_0) = \text{const}$:

$$\Delta f_i = \Delta f_i^0 + \Delta f_i .$$

If we expand φ_i and ψ_i as Taylor series in $\delta\sigma$ and take only the linear members (as $\delta\sigma$ is small), we obtain:

$$\tau_i \frac{dx_i}{dt} + x_i = \lambda_i \delta\sigma ; \tag{2}$$

$$x = \sum_{i=1}^{n} x_i , \tag{3}$$

where x_i is for δf_i, and x – is for δf.

Then

$$\lambda_i = \frac{1}{\varphi_i}\left(\frac{d\psi_i}{d\sigma} - \Delta f_i^0 \frac{d\varphi_i}{d\sigma}\right)\Bigg|_{\sigma=\sigma_0} ; \; \tau_i = \frac{1}{\varphi_i}\Bigg|_{\sigma=\sigma_0} .$$

In case of stepwise change of stress $\delta\sigma = \sigma_1 F(t)$ the solutions of (2) and (3) are:

$$1 - \frac{x}{x_c} = \sum_{i=1}^{n} g_i \exp\left(-\frac{t}{\tau_i}\right); \sum_{i=1}^{n} g_i = 1 , \tag{4}$$

where $x_c = \lim_{i\to\infty} x_i g_i = \dfrac{\lambda_i}{\sum_{i=1}^{n}\lambda_i}$.

Function g_i is the weight of the i – th process. If $n \to \infty$, then g_i becomes infinitely small and the equation (4) now becomes

$$1 - \frac{x}{x_c} = \int_0^\infty g(\tau)\exp\left(-\frac{t}{\tau}\right)d\tau; \int_0^\infty g(\tau)d\tau = 1 , \tag{5}$$

where $g(\tau)$ is the probability density function.

All the processes which cause the changes of the elastic modulus or the internal friction may be considered as inertialess (i.e. practically very short, having low activation energy) and inertial (viscous, with high activation energy). If we denote the relative weight of the first-type processes with the dimensionless parameter ξ ($\xi \le 1$), then we obtain from (5):

$$\eta(t) = 1 - \frac{x}{x_c} = (1-\xi)\int_0^\infty g(\tau)\exp\left(-\frac{t}{\tau}\right)d\tau , \tag{6}$$

where τ – is the relaxation time.

As the stress σ_0 increases, the relative weight of the dimensionless processes also increases. It means, that the function $\xi(\sigma_0)$ is a nondecreasing one. Now we need to fit the function $g(\tau)$ to the experimental curves. One can show that this function must satisfy an integral equation:

$$\int_0^\infty g(\tau)\exp\left(-\frac{t}{\tau}\right)d\tau = \frac{1}{1+Ct} .$$

Its solution is as follows:

$$g(\tau) = \frac{1}{c\tau^2}\exp\left(-\frac{1}{c\tau}\right).$$

The most probable relaxation time, when $g(t)$ is maximal, is the following:

$$\tau_m = \frac{1}{2B} ,$$

and the average relaxation time is:

$$\bar{\tau} = \int_0^\infty \tau g(\tau)d\tau .$$

Experimental curves are well fitted by the function linear from time within some interval and may be described using the following function $g(\tau)$:

$$g(t) = \frac{1}{\tau(\ln\tau_{\max} - \ln\tau_{\min})} \quad \text{if} \quad \tau_{\min} < \tau < \tau_{\max};$$
$$g(t) = 0 \quad \text{if} \quad \tau < \tau_{\min}; \tau > \tau_{\max}.$$

Then we obtain:

$$\frac{x}{x_0} = 1 + \frac{1-\xi}{\ln\tau_{\max} - \ln\tau_{\min}}\left[E_i\left(-\frac{t}{\tau_{\max}}\right) - E_i\left(-\frac{t}{\tau_{\min}}\right)\right],$$

where E_i – is the integral exponential function:

$$E_i\left(-\frac{t}{\tau}\right) = -\int_{-\frac{t}{\tau}}^{\infty} \frac{\exp\left(\frac{t}{\tau}\right)}{t}dt = C + \ln\frac{t}{\tau} + \sum_{k=1}^{\infty}(-1)^k \frac{\left(\frac{t}{\tau}\right)^k}{k \times k!}; \tag{7}$$

$C = 0{,}57721\ldots$ – is the Euler constant.

For very small values of time, less than the low limit of the relaxation times spectrum ($t \ll \tau_{\min}$), one can obtain using (7):

$$-\frac{x}{x_0} \approx \frac{1-\xi}{(\ln\tau_{\max} - \ln\tau_{\min})}\left(\frac{1}{\tau_{\min}} - \frac{1}{\tau_{\max}}\right)t + \xi .$$

It means, that the parameter x increases (or decreases) according to the linear law.

If the interval ($\tau_{\min}$, $\tau_{\max}$) is sufficiently wide, then in some range we have: $\tau_{\min} \ll t \ll \tau_{\max}$. If $(t / \tau) \gg 1$, then

$$E_i\left(-\frac{t}{\tau}\right) \approx \frac{\exp\left(-\frac{t}{\tau}\right)}{\frac{t}{\tau}}\left(1-\frac{1}{\frac{t}{\tau}}+\frac{2!}{\left(\frac{t}{\tau}\right)^2}-\ldots\right) \to 0.$$

And we obtain the logarithmic law:

$$\frac{x}{x_c} \approx 1+\frac{1-\xi}{\ln\tau_{\max}-\ln\tau_{\min}}(C-\ln\tau_{\max}+\ln t).$$

At last, for the values of time exceeding the upper limit of time relaxation spectrum ($t >> \tau_{max}$), if

$$E_i\left(-\frac{t}{\tau_{\min}}\right)=0;$$

$$E_i\left(-\frac{t}{\tau_{\max}}\right) \approx -\frac{\exp\left(-\frac{t}{\tau_{\max}}\right)}{t}\tau_{\max},$$

The time dependence is the following:

$$\frac{x}{x_c} \approx 1-\frac{\tau_{\max}(1-\xi)}{\ln\tau_{\max}-\ln\tau_{\min}}\frac{\exp\left(-\frac{t}{\tau_{\max}}\right)}{t},$$

And is in good correspondence with the experiment.

Now it is possible to evaluate the lower and the upper limits of the time relaxation spectrum. The method is clear from the figure 4. For this type of material (Khibinite) at the room temperature (about 300 K) $\tau_{min} = 20$ sec, $\tau_{max} = 4 \cdot 10^5$ sec. The values of the activation energy can be obtained by measuring the temperature dependences of these spectrum limits.

The detailed analysis of all possible physical processes in porous and microcracking polycrystalline rocks is far out of scope of this paper.

5. The Accuracy of Measurements

The accuracy of measurements of the porosity and the internal friction is rather low. The porosity may be measured using the pyknometer and the accuracy does not exceed 0.02, or using the method of hydrostatic weighting with the accuracy of about 0.001. But the average dispersion for different specimens may be as high as 80 – 90%. The accuracy of measurements of the internal friction is about 0.04 – 0.05 and the dispersion for different specimens do not exceed 2 – 3%.

The relative changes of the dynamic Young's modulus are measured by the described ultrasonic method, and these measurements are much more accurate. One must mind that the Young's modulus depends mainly from the mineral structure of rock and not only from the presence of waste phase. That is why it is only possible to compare the relative changes of the Young's modulus (the Young's modulus defect) under the influence of the external action, i.e. the uniaxial constant stress. Figure 2 shows the dependences of the Young's modulus defect from stress up to 140 MPa for different rocks. The stress was below the threshold of destruction.

In general we can conclude that ultrasonic methods are much more accurate and acceptable to stress measurements than other methods. Among other ultrasonic methods one can also use the pulse and pulse-interference methods [44], [45]. They are valuable, very accurate and especially valid for measurements of anisotropy and changes of anisotropy under stress. Still, the resonance ultrasonic method of loaded complex oscillator remains the most convenient method to precise study of creep [45].

6. Conclusion

The paper shows the advantages of the ultrasonic method of loaded complex oscillator and the possibility of its usage to study dependences of the Young's modulus and of the internal friction from constant uniaxial stress. Different porous and microcrambling polycrystalline rocks with different levels of micro-porosity and microcrublance from khibinite – the most porous up to granulite – the least porous were studied. These dependences were obtained for the first time, as well as for granite, gabbro, dolomite and quartzite. The data may become useful for practical usage, for example for the acoustical waterproofing treatment of mine openings and building structures [46].

The resonance loaded complex oscillator method was proposed and tested to study the elastic creep for practically unlimited time of being under load.

The phenomenological model of creep was also proposed for different rocks. We did not discuss physical mechanisms, which take place in these materials under such conditions. This may be the subject of the further studies. The proposed method may become a valuable addition to existing geophysical methods of solid state study.

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