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On Eigenvalues of Nonlinear Operator Pencils with Many Parameters

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Abstract

The authors give the necessary and sufficient conditions of the existence of the common eigenvalues of the nonlinear several operator pencils with many parameters. The operator pencils contain also the products of these parameters in finite degree. The number of equations in these systems may be more, than the number of parameters. In the proof the authors essentially use the results of multiparameter spectral theory and the notion of the analog resultant of two and several operator pencils in many parameters.

Keywords

Resultant, Operator, Multiparameter System, Eigenvalue

1. Introduction

Spectral theory of operators is one of the important directions of functional analysis. The development of physical sciences becoming more and more challenges to mathematicians. In particular, the resolution of the problems associated with the physical processes and, consequently, the study of partial differential equations and mathematical physics equations, required a new approach. The method of separation of variables in many cases turned out to be the only acceptable, since it reduces finding a solution of a complex equation with many variables to find a solution of a system of ordinary differential equations, which are much easier to study.

2. Necessary Definitions and Remarks

Give some definitions and concepts from the theory of multiparameter operator systems necessary for understanding of the further considerations.

Let be the linear multiparameter system in the form:

$$B_k(\lambda)x_k = (B_{0,k} + \sum_{i=1}^{n} \lambda_i B_{i,k})x_k = 0, \quad k = 1,2,...,n \tag{1}$$

when operators $B_{k,i}$ act in the Hilbert space H_i

Definition 1. [1,2,11] $\lambda = (\lambda_1, \lambda_2, ..., \lambda_n) \in C^n$ is an eigenvalue of the system (1) if there are non-zero elements $x_i \in H_i$, $i = 1,2,...,n$ such that (1) satisfy, and decomposable tensor $x = x_1 \otimes x_2 \otimes ... \otimes x_n$ is called the eigenvector corresponding to eigenvalue $\lambda = (\lambda_1, \lambda_2, ..., \lambda_n) \in C^n$.

Definition 2. The operator $B_{s,i}^{+}$ is induced by an operator $B_{s,i}$, acting in the space H_i , into the tensor space $H = H_1 \otimes ... \otimes H_n$, if on each decomposable tensor $x = x_1 \otimes ... \otimes x_n$ of tensor product space $H = H_1 \otimes ... \otimes H_n$ we have $B_{s,i}^{+} x = x_1 \otimes ... \otimes x_{i-1} \otimes B_{s,i} x_i \otimes x_{i+1} \otimes ... \otimes x_n$ and on all the other elements of $H = H_1 \otimes ... \otimes H_n$ the operator $B_{s,i}^{+}$ is defined on linearity and continuity.

Definition 3 ([5], [6]).

Let $x_{0,...,0} = x_1 \otimes x_2 \otimes ... \otimes x_n$ be an eigenvector of the system (1), corresponding to its eigenvalue $\lambda = (\lambda_1, \lambda_2, ..., \lambda_n)$; the $x_{m_1,...,m_n}$ is $m_1, m_2, ..., m_n$ - associated vector (see[4]) to an eigenvector $x_{0,0,\dots,0}$ of the system (1) if there is a set of vectors $\{x_{i_1,i_2,\dots,ii_n}\} \subset H_1 \otimes \cdots \otimes H_n$, satisfying to conditions

$$B^+_{0,i}(\lambda)x_{is,s_2,\dots,s_n} + B^+_{1,i}x_{s_1-1,s_2,\dots,s_n} + \dots + B^+_{n,i}x_{s_1,\dots,s_{n-1},s_n-1} = 0 .$$

$$x_{i_{s1},s_2,\dots,s_n} = 0, \text{ when } s_i < 0 \quad (2)$$

$$0 \le s_r \le m_r,\ r = 1,2,\dots,n,\ i = 1,\dots,n$$

Indices $s_1, s_2, \dots, s_n$ in element $(x_{i_1,i_2,\dots,i_n}) \subset H_1 \otimes \cdots \otimes H_n$ there are various arrangements from set of integers on n with $0 \le s_r \le m_r$, $r = 1,2,\dots,n$, .

Definition 4. In [1,3,11] for the system (1) analogue of the Cramer's determinants, when the number of equations is equal to the number of variables, is defined as follows: on decomposable tensor $x = x_1 \otimes \dots \otimes x_n$ operators Δ_i are defined with help the matrices

$$\sum_{i=0}^{n} \alpha_i \Delta_i x = = \otimes \begin{pmatrix} \alpha_0 & \alpha_1 & \alpha_2 & \dots & \alpha_n \\ B_{0,1}x_1 & B_{1,1}x_1 & B_{2,1}x_1 & \dots & B_{n,1}x_1 \\ B_{0,2}x_2 & B_{1,2}x_2 & B_{2,2}x_2 & \dots & B_{n,2}x_2 \\ B_{0,3}x_3 & B_{1,3}x_3 & B_{2,3}x_3 & \dots & B_{n,3}x_3 \\ \dots & \dots & \dots & \dots & \dots \\ B_{0,n}x_n & B_{1,n}x_n & B_{2,n} & \dots & B_{n,n} \end{pmatrix} \quad (3)$$

where $\alpha_0, \alpha_1, \dots, \alpha_n$ are arbitrary complex numbers, under the expansion of the determinant means its formal expansion, when the element $x = x_1 \otimes x_2 \otimes \dots \otimes x_n$ is the tensor products of elements $x_1, x_2, \dots, x_n$ If $\alpha_k = 1, \alpha_i = 0, i \ne k$, ,then right side of (10) equal to $\Delta_k x$, where $x = x_1 \otimes x_2 \otimes \dots \otimes x_n$ On all the other elements of the space H operators Δ_i are defined by linearity and continuity. $E_s (s = 1,2,\dots,n)$ is the identity operator of the space H_i .Suppose that for all $x \ne 0$, $(\Delta_0 x, x) \ge \delta(x,x)$, $\delta > 0$, and all $B_{i,k}$ are selfadjoint operators in the space H_i . Inner product [.,.] is defined as follows; if $x = x_1 \otimes x_2 \otimes \dots \otimes x_n$ and $y = y_1 \otimes y_2 \otimes \dots \otimes y_n$ are decomposable tensors, then $[x, y] = (\Delta_0 x, y)$ where (x_i, y_i) is the inner product in the space H_i .On all the other elements of the space H the inner product is defined on linearity and continuity. In space H with such a metric all operators $\Gamma_i = \Delta_0^{-1}\Delta_i$ are selfadjoint

Definition5.([7],[8])

Let be two operator pencils depending on the same parameter and acting in, generally speaking, in various Hilbert spaces

$$A(\lambda) = A_0 + \lambda A_1 + \lambda^2 A_2 + \dots + \lambda^n A_n,$$
$$B(\lambda) = B_0 + \lambda B_1 + \lambda^2 B_2 + \dots + \lambda^m B_m$$

Operator $\operatorname{Re} s(A(\lambda), B(\lambda))$ is presented by the matrix

$$\begin{pmatrix} A_0 \otimes E_2 & A_1 \otimes E_2 & \dots & A_n \otimes E_2 & \dots & 0 \\ . & . & \dots & . & \dots & . \\ 0 & 0 & \dots A_0 \otimes E_2 & A_1 \otimes E_2 & \dots & A_n \otimes E_2 \\ E_1 \otimes B_0 & E_1 \otimes B_1 & \dots & E_1 \otimes B_m & .. & 0 \\ . & . & \dots & . & \dots & . \\ . & . & \dots E_1 \otimes B_0 & E_1 \otimes B_1 & \dots & E_1 \otimes B_m \end{pmatrix} \quad (4)$$

which acts in the $(H_1 \otimes H_2)^{n+m}$ - direct sum of $n+m$ copies of the space $H_1 \otimes H_2$ In a matrix (4). number of rows with operators A_i is equal to leading degree of the parameter λ in pencils $B(\lambda)$ and the number of rows with B_i is equal to the leading degree of parameter λ in $A(\lambda)$. Notion of abstract analog of resultant of two operator pencils is considered in the[7] for the case of the same leading degree of the parameter in both pencils and in the [2]for, generally speaking, different degree of the parameters in the operator pencils.

Theorem1 [7,8].

[Let all operators are bounded in corresponding Hilbert spaces, one of operators A_n or B_m has bounded inverse Then operator pencils $A(\lambda)$ and $B(\lambda)$ have a common point of spectra if and only if

$$Ker \operatorname{Re} s(A(\lambda), B(\lambda)) \ne \{\vartheta\}$$

Remark1. If the Hilbert spaces H_1 and H_2 are the finite dimensional spaces then a common points of spectra of operator pencils $A(\lambda)$ and $B(\lambda)$ are their common eigenvalues. (see [6], [7].)

Consider n bundles depending on the same parameter λ

$$\{B_i(\lambda) = B_{0,i} + \lambda B_{1,i} + \dots + \lambda^{k_i} B_{k_i,i}, \quad i = 1,2,\dots,n$$

$B_i(\lambda)$ - operator bundles acting in a finite dimensional Hilbert space H_i correspondingly. Without loss of generality. we adopt $k_1 \ge k_2 \ge \dots \ge k_n$. In the space $H^{k_1+k_2}$ (the direct sum of $k_1 + k_2$ tensor product $H = H_1 \otimes \dots \otimes H_n$ of spaces $H_1, H_2, \dots, H_n$) are introduced the operators R_i $(i = 1,\dots,n-1)$ with the help of operational matrices (3.12) Let $B_i(\lambda)$ be the operational bundles acting in a finite dimensional Hilbert space H_i .

$$R_{i-1} = \begin{pmatrix} B^+_{0,1} & B^+_{1,1} & \cdots & B^+_{k_1,1} & \cdots & 0 \\ 0 & B^+_{0,1} & B^+_{1,1} \cdot\cdot & B^+_{k_1-1.1} & B^+_{k_1,1} \cdot\cdot & 0 \\ . & . & \cdot\cdot & . & \cdots & . \\ 0 & 0 & \cdot\cdot B^+_{0,1} & B^+_{1,1} & \cdots & B^+_{k_1,1} \\ B^+_{0,i} & B^+_{1,i} & \cdots & B^+_{k_i,i} & 0.\cdot\cdot & 0 \\ 0 & B^+_{0,i} & B^+_{1,i} \cdots & \cdot & B^+_{k_i,i} \cdot\cdot & 0 \\ . & . & \cdots & . & \cdots & . \\ 0 & 0 & \cdots B^+_{0,i} & B^+_{1,i} & \cdots & B^+_{k_i,i} \end{pmatrix},$$

$$i = 2,3,...,n$$

The number of rows with operators $B_{s,1}, s = 0,1,...,k_1$ in the matrix R_{i-1} is equal to k_2 and the number of rows with operators $B_{s,i}, s = 0,1,...,k_i$ is equal to k_1. We designate $\sigma_p(B_i(\lambda))$ the set of eigenvalues of an operator $B_i(\lambda)$ .From [5] we have the result:

Theorem 2.[9] $\bigcap_{i=1}^{n}\sigma_p(B_i(\lambda)) \neq \{\theta\}$ if and only if $\bigcap_{i=1}^{n-1} KerR_i \neq \{\theta\}$, $(KerB_{k_1} = \{\theta\})$.

3. The System of Operator Pencils in Many Variables

Consider the system

$$A_{i,j,s}(\lambda)x_s = (A_{0,s} + \sum_{r=1}^{k_{1,s}}\lambda_1^r A_{1,r,s} + ... + \sum_{r=1}^{k_{n,s}}\lambda_n^r A_{n,r,s} + \\ + \sum \lambda_1^{i_1}\lambda_2^{i_2}...\lambda_n^{i_n} A_{i_1,...,i_n} \\ s = 1,2,...,n \tag{5}$$

The parameters $\lambda_1, \lambda_2, ..., \lambda_n$ enter the system nonlinearly, and the system (4) contains also the products of these parameters. Divide the system of equations (5) into groups of n in each group. If some equations remains outside, these equations we add by others operators from the system (4). Each group contains n operators and will be considered separately.

In (5) the coefficients of the parameter λ_m^r, $r \le k_m$, $m = 1,2,...,n$ are the operators $A_{i,m,j}$, which act in the space H_j , index i indicate on the parameter λ_i , index k - on the degree of the parameter λ_i .

Introduce the notations:

$$\lambda_m^r = \lambda_{k_1+k_2+...+k_{m-1}+r}, \ r \le k_m, \ \ m = 1,2,...,n \tag{6}$$

Further , numerate the different products of variables $\lambda_1, \lambda_2, ..., \lambda_n$ in the system (5) on increasing of the degrees of the parameter λ_1 . Let the numbers of term with the products of the parameters $\lambda_1, \lambda_2, ..., \lambda_n$ are equal to r .

Introduce the notations

$$\lambda_1^{i_1}\lambda_2^{i_2}...\lambda_n^{i_n} = (\lambda_1^{i_1}\lambda_2^{i_2}...\lambda_n^{i_n})_t = \tilde{\lambda}_{k_1+k_2+...+k_n+t}, \ t \le r,$$

where $t \le s$ is the number which correspond the multiplier at $\lambda_1^{i_1}\lambda_2^{i_2}...\lambda_n^{i_n}$ the ordering of multiplies of parameters in the system (5). So in new notations to the product $\lambda_1^{i_1}\lambda_2^{i_2}...\lambda_n^{i_n}$ correspond the parameter $\tilde{\lambda}_{k_1+k_2+...+k_n+t}$, $t \le r$ ($\lambda_1^{i_1}\lambda_2^{i_2}...\lambda_n^{i_n} \to \tilde{\lambda}_{k_1+k_2+...+k_n+t}$, $t \le r$), accordingly, operators

$$A_{r,s,i} = D_{k_1+k_2+...+k_{s-1}+s,i}, r = 1,2,...,n; s = 1,2,...,k_r; \\ i = 1,2,...,n$$

$$k_r = \max k_{r,i}, i = 1,2,...,k \ , \tag{7}$$

$$A_{k_1,k_2...,k_n;i} = D_{k_1+k_2+...+k_m+t,i}, t = 1,2,...,s;; i = 1,2,...,n$$

when s is the number of different products of parameters, entering the system(5).

In new notations the system (5) in the tensor product of spaces $H_1 \otimes H_2 \otimes ... \otimes H_n$ contains $k_1 + k_2 + ... + k_n + s$ parameters and n equations. Let $k_1 + k_2 + ... + k_n = k$ Then

$$\sum_{r=0}^{n}\sum_{k=1}^{k_r}[\tilde{\lambda}_{k_1+k_2+...+k_{r-1}+k} D_{k_1+k_2+...+k_{r-1}+k,i}]x_i + [\sum_{k=1}^{r}\tilde{\lambda}_{k+t} D_{k+t,i}]x_{i=0} = 0 \\ k_0 = 0; \ k_{-i} = 0; \ i = 1,2,...,n \tag{8}$$

Add the system (8) with help of new equations so manner that the connections between the parameters, following from the equations of the system (5), satisfy. Introduce the operators $T_0, T_1, T_2, \overline{T_0}, \overline{\overline{T}}_0$ acting in the finite dimensional space R^2 and defining with help of the matrices

$$T_0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \ T_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \ T_2 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \ \overline{\overline{T}}_0 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix},$$

$$T_{1,s_1,r} = \begin{pmatrix} 1 & 0 & 0 & ... & 0 & 0 & 0 \\ 0 & 1 & 0 & ... & 0 & 0 & 0 \\ 0 & 0 & 1 & ... & 0 & 0 & 0 \\ 0 & 0 & 0 & ... & 0 & 0 & 0 \\ . & . & . & ... & . & . & . \\ 0 & 0 & 0 & ... & 0 & 0 & 0 \\ 0 & 0 & 0 & ... & 0 & 0 & 0 \end{pmatrix}, \ldots, T_{k_n+1,s_n,r} = \begin{pmatrix} 0 & 0 & 0 & ... & 0 & 0 & 0 \\ 0 & 0 & 0 & ... & 0 & 0 & 0 \\ 0 & 0 & 0 & ... & 0 & 0 & 0 \\ 0 & 0 & 0 & ... & 0 & 0 & 0 \\ . & . & . & ... & . & . & . \\ 0 & 0 & 0 & ... & 0 & 1 & 0 \\ 0 & 0 & 0 & ... & 0 & 0 & 1 \end{pmatrix}$$

$$T_{(s_1,s_2,...,s_n)_r} = \begin{pmatrix} 0 & 0 & 0 & ... & 0 & 0 & 1 \\ 1 & 0 & 0 & ... & 0 & 0 & 0 \\ 0 & 1 & 0 & ... & 0 & 0 & 0 \\ 0 & 0 & 1 & ... & 0 & 0 & 0 \\ . & . & . & ... & . & . & . \\ 0 & 0 & 0 & ... & 1 & 0 & 0 \\ 0 & 0 & 0 & ... & 0 & 1 & 0 \end{pmatrix} \tag{9}$$

The number1 stands on the diagonal elements of the first s_1 rows of the matrix $T_{1,s_1,r}$; diagonal elements of the rows $s_1 + s_2 + ... + s_{i-1} + 1, ..., s_1 + s_2 + ... + s_i$ of the matrix $T_{k_i+1,s_{i+1},r}$ is equal also to 1 and so on. Besides, all matrices $T_{1,s_1,r}, ..., T_{k_i+1,s_{i+1},r}, ..., T_{(s_1,s_2,...,s_n)_r}$ have the order $s_1 + s_2 + ... + s_n$.

Add the system (8) by the following equations

$$(T_{2,n+1} + \tilde{\lambda}_1 T_{0,n+1} + \tilde{\lambda}_2 T_{1,n+1})x_{n+1} = 0$$
$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$
$$(\tilde{\lambda}_{k_1+k_2-2} T_{2,n+k_1+k_2-2} + \tilde{\lambda}_{k_1+k_2-1} T_{0,n+k_1+k_2-2} + $$
$$+\tilde{\lambda}_{k_1+k_2} T_{1,n+k_1+k_2-2})x_{n+k_1+k_2-2} = 0$$
$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$
$$(\tilde{\lambda}_{k_1+\dots+k_{n-1}-2} T_{2,1+\sum_{i=1}^{n-1} k_i} + \tilde{\lambda}_{k_1+\dots+k_{n-1}-1,0} T_{0,1+\sum_{i=1}^{n-1} k_i})x_{n+k_1+\dots+k_{n-1}-2} = 0$$
$$(\tilde{\lambda}_{k_1+\dots+k_n-2} T_2 + \tilde{\lambda}_{k_1+\dots+k_n-1} T_0 + \tilde{\lambda}_k T_1)x_k = 0$$
$$x_s \in R^2,\ s > n \tag{10}$$
$$(T_{0,t} + \tilde{\lambda}_1 T_{i_1,t} + \tilde{\lambda}_{k_1+1} T_{i_2,t} + \dots + \tilde{\lambda}_{k_1+\dots+k_{n-1}+1} T_{i_n,t} -$$
$$-\tilde{\lambda}_{k+(i_1,i_2,\dots,i_n)_t} T_{(i_1,i_2,\dots,i_n)})x_t = 0$$
$$t = 1,2,\dots,r$$

Denote $\tilde{\lambda}_{k+(i_{11},i_2,\dots,i_n)_r}$ the multiplier $\lambda_1^{i_1}\lambda_2^{i_2}\dots\lambda_n^{i_n}$ of the parameters, entering the system (5) having the coefficient $A_{(i_{11},i_2,\dots,i_n)_r}$. System ((5),(10)) form the linear multiparameter system, containing $k_1 + k_2 + \dots + k_n + r$ equations and $k_1 + k_2 + \dots + k_n + s$ parameters. To this system we may apply all results, given in the beginning of this paper.

Theorem 1. [4]. Let the following conditions:

a) operators $A_{k,t}, A_{k_1,k_2,\dots,k_n;t}$ in the space H_i are bounded at the all meanings i and k.

b) operator Δ_0^{-1} exists and bounded satisfy:

Then the system of eigen and associated vectors of (5) coincides with the system of eigen and associated vectors of each operators $\Gamma_i (i = 1,2,\dots,n)$

Give two equations from (10). Let be equations

$$\begin{aligned}(T_2 + \lambda_1 T_0 + \lambda_2 T_1)x_{n+1} = 0 \\ (\lambda_1 T_2 + \lambda_2 T_0 + \lambda_3 T_1)x_{n+2} = 0\end{aligned} \tag{11}$$

Let $\lambda_1 \neq 0$ и $x_{n+1} = (\alpha_1, \beta_1) \neq 0$ is the component of the eigenvector of the system ((5),(10)). We have

$$\left(\begin{pmatrix}0 & 0\\0 & 1\end{pmatrix} + \lambda_1\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix} + \lambda_2\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}\right)(\alpha_1, \beta_1) = 0,$$

$$\lambda_1\beta_1 + \lambda_2\alpha_1 = 0,\ \beta_1 + \lambda_1\alpha_1 = 0,\ \lambda_2 \neq 0; \lambda_2 = \lambda_1^2.$$

Further from the condition $\lambda_1 \neq 0, \lambda_2 \neq 0, x_{n+2} = (\alpha_2, \beta_2) \neq 0$ it follows $\lambda_2\beta_2 + \lambda_3\alpha_2 = 0$, $\lambda_1\beta_2 + \lambda_2\alpha_1 = 0$ and consequently, $\tilde{\lambda}_1\tilde{\lambda}_3 = \tilde{\lambda}_2^2$. Earlier we proved that $\tilde{\lambda}_2 = \lambda_1^2$, Consequently, $\tilde{\lambda}_3 = \lambda_1^3$.

On analogy for other parameters of ((5),(10)): if $(\tilde{\lambda}_1, \tilde{\lambda}_2, \dots, \tilde{\lambda}_{k_1+k_2+\dots+k_n+s})$ is the eigenvalue of the system -((5), (10)) , then $\lambda_4 = \lambda_1^4$, …, $\lambda_{k_1} = \lambda_1^{k_1}$, … , $\tilde{\lambda}_{k_1+k_2+\dots+k_r+s} = \lambda_{r+1}^s$, $r = 1,2,\dots,n-1; s = 1,2,\dots,k_n$.

To each multiplier of parameters $(\tilde{\lambda}_{j_1}^{r_{j_1}}\tilde{\lambda}_{j_2}^{r_{j_2}}\cdots\tilde{\lambda}_{j_k}^{r_{j_k}})_t = \tilde{\lambda}_{k+t}$; $t \le r$ it is corresponded the equation

$$(T_{0,t+k} + \tilde{\lambda}_1 T_{1,i_1,k+t} + \tilde{\lambda}_{k_1+1} T_{2,i_2,k+t} + \dots + \tilde{\lambda}_{k_1+k_2+\dots+k_{n-1}+1} T_{n,i_n,k+t} -$$
$$-\tilde{\lambda}_{k+(i_1,i_2,\dots,i_n)_t} T_{(i_1,\dots,i_n)_t,t+k})x_{k+t} = 0$$

Consider the last equation, in which

$$T_{1,s_1,r} = \begin{pmatrix} 1 & 0 & 0 & \dots & 0 & 0 & 0\\ 0 & 1 & 0 & \dots & 0 & 0 & 0\\ 0 & 0 & 1 & \dots & 0 & 0 & 0\\ 0 & 0 & 0 & \dots & 0 & 0 & 0\\ . & . & . & \dots & . & . & .\\ 0 & 0 & 0 & \dots & 0 & 0 & 0\\ 0 & 0 & 0 & \dots & 0 & 0 & 0 \end{pmatrix},$$

$$T_{k_1+\dots+k_{n-1}+1,s_n,r} = \begin{pmatrix} 0 & 0 & 0 & \dots & 0 & 0 & 0\\ 0 & 0 & 0 & \dots & 0 & 0 & 0\\ 0 & 0 & 0 & \dots & 0 & 0 & 0\\ 0 & 0 & 0 & \dots & 0 & 0 & 0\\ . & . & . & \dots & . & . & .\\ 0 & 0 & 0 & \dots & 0 & 1 & 0\\ 0 & 0 & 0 & \dots & 0 & 0 & 1 \end{pmatrix}$$

$$\tilde{T}_{(s_1,\dots,s_n)_t,r} = \begin{pmatrix} 0 & 0 & 0 & \dots & 0 & 0 & 1\\ 0 & 0 & 0 & \dots & 0 & 0 & 0\\ 0 & 0 & 0 & \dots & 0 & 0 & 0\\ 0 & 0 & 0 & \dots & 0 & 0 & 0\\ . & . & . & \dots & . & . & .\\ 0 & 0 & 0 & \dots & 0 & 0 & 0\\ 0 & 0 & 0 & \dots & 0 & 0 & 0 \end{pmatrix}$$

$$\tilde{T}_{0,r} = \begin{pmatrix} 0 & 0 & 0 & \dots & 0 & 0 & 0\\ 1 & 0 & 0 & \dots & 0 & 0 & 0\\ 0 & 1 & 0 & \dots & 0 & 0 & 0\\ 0 & 0 & 1 & \dots & 0 & 0 & 0\\ . & . & . & . & . & . & .\\ 0 & 0 & 0 & \dots & 1 & 0 & 0\\ 0 & 0 & 0 & \dots & 0 & 1 & 0 \end{pmatrix}$$

Operators, defining with help the matrices $T_{1,s_1,k+t}, T_{2,s_2,k+t}, \dots, T_{n,s_n,k+t}, T_{0.k+t}$ act in space $R^{s_1+\dots+s_n}$ On eigenvector $(\alpha_1, \dots, \alpha_{s_1+s_2+\dots+s_n}) \in R^{s_1+\dots+s_n}$ equations .

$$(-\tilde{T}_{0,r} + \tilde{\lambda}_1 T_{1,s_1,r} + \dots + \tilde{\lambda}_{1+\sum_{i=1}^{n-1} k} T_{1+k_1+\dots+k_{n-1},s_n,r})\tilde{\alpha} =$$
$$= \tilde{\lambda}_{k+t} T_{(s_1\dots s_{nt}} r)\tilde{\alpha}$$

satisfy.

Consequently,

$$\tilde{\lambda}_1 \alpha_1 = \tilde{\lambda}_{k+t} \alpha_{s_1+\ldots+s_n}$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$\tilde{\lambda}_1 \alpha_{s_1} = \alpha_{s_1-1}$$

$$\tilde{\lambda}_{k_1+1} \alpha_{s_1+1} = \alpha_{s_1}$$

$$\ldots\ldots\ldots\ldots\ldots$$

$$\tilde{\lambda}_{k_1+1} \alpha_{s_1+s_2} = \alpha_{s_1+s_2-1}$$

$$\ldots\ldots\ldots\ldots\ldots\ldots$$

$$\tilde{\lambda}_{1+\sum_{i=1}^{n-1} k_i} \alpha_{s_1+s_2+\ldots+s_{1+\sum_{i=1}^{n-1} k_i}} = \alpha_{s_1+s_2+\ldots+s_{1+\sum_{i=1}^{n-1} r_i}-1}$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

Last means $\lambda_1^{s_1} \lambda_2^{s_2} \cdots \lambda_n^{s_n} = \lambda_{k+s};\ \ s \le r.$

For the obtained linear multiparameter system we construct operator Δ_0 on rule (3).

The condition $Ker\Delta_0^{-1} = \{\vartheta\}$ means that operators $\Gamma_i = \Delta_0^{-1}\Delta_i$ are pair commute[2]. So operators Γ_i act in finite dimensional space H and operators $\Gamma_{k_1+k_2+\ldots+k_{r-1}+1}$ have not the zero eigenvalues then for the any eigenvalue $(\lambda_1, \lambda_2, \ldots, \lambda_{k_1+k_2+\ldots+k_n})$ of the system((5), (10))

$\Delta_{k_1+k_2+\ldots+k_{i-1}+1,i} z_i = \lambda_{i,s} \Delta_{o,i} z_i$ there is such eigen element z that the equalities,

$\Gamma_i z = \lambda_i z;\ \ i = 1, 2, \ldots, k_1 + k_2 + \ldots + k_n + s;$ satisfy. For analogy conditions we obtain the analogy results for all groups. We have the several systems of operator polynomials in one parameter.

The system has the form

$$\Delta_{i,1} z_1 = \lambda_i \Delta_{0,1} z_1$$

$$\Delta_{i,2} z_2 = \lambda_i \Delta_{0,2} z_2$$

$$\ldots\ldots\ldots\ldots\ldots\ldots$$

$$\Delta_{i,m} z_l = \lambda_i \Delta_{0,m} z_m$$

Where $i = 1, 2, \ldots, k_1 + k_2 + \ldots + k_n + s;$ m is the number of groups. We apply the results of [9](theorem2 in this paper)

Theorem 3. Let the conditions of the theorem1 is fulfilled. All operators $\Delta_{o,i}$ have inverse. Then the system (4) has the common eigenvalue if and only if

$$Ker \bigcap_{t=1}^{m} (\Delta^{+}_{k_1+k_2+\ldots+k_{i-1}+1,t} - \lambda_i \Delta^{+}_{0,t}) \neq 0$$

$$i = 1, 2, \ldots, n.$$

We apply the results of [9](theorem2 in this paper)

Operators $\Delta^{+}_{r,t}$ are induced into the space $H_1 \otimes \ldots \otimes H_l$ by the operators $\Delta_{r,t}$, correspondingly.

Theorem 3. Let the conditions of the theorem1 is fulfilled. All operators $\Delta_{o,i}$ have inverse. Then the system(4) has the common eigenvalue if and only if

$$Ker \bigcap (\Delta_{k_1+k_2+\ldots+k_{i-1}+1,i} - \lambda_{i,s} \Delta_{o,i}) \neq 0$$

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