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Fuzzy Optimization Approach to Economic Order Quantity (FEOQ) Model for Deteriorating Items Including Variable Ordering Cost and Promotional Effort Cost

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Abstract

The instantaneous economic order quantity stylized model is introduced for analyzing the effect of variable ordering cost and promotional effort cost with deteriorated items. The objective of this model is to maximize the net profit so as to determine the order quantity in the fuzzy decision space. For any given number of replenishment cycles the existence of a unique optimal replenishment schedule are proved and further the concavity of the net profit function of the inventory system in the number of replenishment is established. The numerical analysis shows that an appropriate policy can benefit the retailer and that policy is important, especially for deteriorating items. Finally, sensitivity analyses of the fuzzy optimal solution with respect to the major parameters are also studied to draw some decisions with managerial insights which are cost effective for competitive advantage in a nonrandom uncertain market.

Keywords

Fuzzy, FEOQ, Variable Ordering Cost, Promotional Effort Cost, Deterioration

1. Introduction

Most of the literature on inventory control and production planning has dealt with the assumption that the demand for a product will continue infinitely in the future either in a deterministic or in a stochastic fashion. This assumption does not always hold true. Inventory management plays a crucial role in businesses since it can help companies reach the goal of ensuring prompt delivery, avoiding shortages, helping sales at competitive prices and so forth. The mathematical modeling of real-world inventory problems necessitates the simplification of assumptions to make the mathematics flexible. However, excessive simplification of assumptions results in mathematical models that do not represent the inventory situation to be analyzed. In the whole production system production function is the mid between the procurement function and physical distribution function. Other two functions are not processing in terms of production only they are facilitating for the smooth functioning and cost effecting of the production system in competitive advantage but production function processes to produce the finished products. So inventory plays a significant role in smooth functioning of the production function in a supply chain management. The physical characteristics of stocked items dictate the nature of inventory policies implemented to manage and control in production system. The question is how reliable are the EOQ models when items stocked deteriorate one time.

Many models have been proposed to deal with a variety of inventory problems. The classical analysis of inventory control considers three costs for holding inventories. These costs are the procurement cost, carrying cost and shortage cost. The classical analysis builds a model of an inventory system and calculates the EOQ which minimize these three costs so that their sum is satisfying minimization criterion. One of the unrealistic assumptions is that items stocked preserve their physical characteristics during their stay in inventory. Items in stock are subject to many possible risks, e.g. damage, spoilage, dryness; vaporization etc., those results decrease of usefulness of the original one and a cost is incurred to account for such risks. Comprehensive reviews of inventory models can be found in Gupta and Gerchak (1995), Osteryoung et al. (1986) and Water (1994) and Tripathy et al. (2013) introduced a single item EOQ model with two constraints. This model considers a continuous review, using fuzzy arithmetic approach to the system cost for instantaneous production process. In traditional inventory models it has been common to apply fuzzy on demand rate, production rate and deterioration rate, whereas applying fuzzy arithmetic in system cost usually ignored in Salameh et al. (1999). From practical experience, it has been found that uncertainty occurs not only due to lack of information but also as a result of ambiguity concerning the description of the semantic meaning of declaration of statements relating to an economic world. The fuzzy set theory was developed on the basis of non-random uncertainties. Vujosevic et al. (1996) introduced the EOQ model where inventory system cost is fuzzy. Mahata and Goswami (2006) then presented production lot size model with fuzzy production rate and fuzzy demand rate for deteriorating items where permissible delay in payments are allowed. Tripathy and Pattnaik (2011) presented an optimal inventory policy with reliability consideration and instantaneous receipt under imperfect production process. Later, Tripathy and Pattnaik (2009, 2011) also investigated fuzzy EOQ model with reliability consideration in instantaneous production plan. Again Tripathy and Pattnaik (2008, 2011) developed fuzzy entropic order quantity model for perishable items under two component demand and discounted selling price, where entropic means the amount of the disorder in the production system. Roy and Maiti (1997) presented fuzzy EOQ model with demand dependent unit cost under limited storage capacity. Pattnaik (2013) discussed the fuzzy EOQ model with demand dependent unit price and variable setup cost, Pattnaik (2011, 2013, 2013) investigated the fuzzy method for supplier selection in manufacturing system for smooth function of supply chain management and manpower selection for micro, small and medium enterprises respectively. For this reason, this model considers the same by introducing the holding cost and ordering cost as with allowing promotion and wasting the percentage of the fuzzy numbers. Sahoo and Pattnaik (2013) developed linear programming problem and post optimality analyses in fuzzy space with case study applications. Pattnaik (2013) defined linear programming problems with crisp and fuzzy based optimization methods and sensitivity analyses have also evaluated for decision parameters. Pattnaik (2013) derived profit maximization fuzzy EOQ models for deteriorating items with two dimension sensitive demand. The model provides an approach for quantifying the benefits of nonrandom uncertainty which can be substantial, and should be reflected in fuzzy arithmetic system cost.

Product perishability is an important aspect of inventory control. Deterioration in general, may be considered as the result of various effects on stock, some of which are damage, decay, decreasing usefulness and many more. While kept in store fruits, vegetables, food stuffs etc. suffer from depletion by decent spoilage. Decaying products are of two types. Product which deteriorate from the very beginning and the products which start to deteriorate after a certain time. Lot of articles is available in inventory literature considering deterioration. Interested readers may consult the survey model of Pattnaik (2011) investigated an entropic order quantity model for perishable items with pre and post deterioration discounts under two component demand in finite horizon. Pattnaik (2011) discussed an economic order quantity model for perishable items with constant demand where instant deterioration discount is allowed to obtain maximum profit. Goyal and Gunasekaran (1995) and Raafat (1991) surveyed for perishable items to optimize the EOQ model. The EOQ inventory control model was introduced in the earliest decades of this century and is still widely accepted by many industries today. Tripathy and Pattnaik (2008, 2011) studied profit maximization entropic order quantity model for deteriorated items with stock dependent demand where discounts are allowed for acquiring more profit. Pattnaik (2012) derived different types of typical deterministic EOQ models in crisp and fuzzy decision space.

Comprehensive reviews of inventory models under deterioration can be found in Bose et al. (1995). In previous deterministic inventory models, many are developed under the assumption that demand is either constant or stock dependent for deteriorated items. Jain and Silver (1994) developed a stochastic dynamic programming model presented for determining the optimal ordering policy for a perishable or potentially obsolete product so as to satisfy known time-varying demand over a specified planning horizon. They assumed a random lifetime perishability, where, at the end of each discrete period, the total remaining inventory either becomes worthless or remains usable for at least the next period. Gupta and Gerchak (1995) examined the simultaneous selection product durability and order quantity for items that deteriorate over time. Their choice of product durability is modeled as the values of a single design parameter that effects the distribution of the time-to-onset of deterioration (TOD) and analyzed two scenarios; the first considers TOD as a constant and the store manager may choose an appropriate value, while the second assumes that TOD is a random variable. Hariga (1995) considered the effects of inflation and the time-value of money with the assumption of two inflation rates rather than one, i.e. the internal (company) inflation rate and the external (general economy) inflation rate. Hariga (1994) argued that the analysis of Bose et al. (1995) contained mathematical errors for which he proposed the correct theory for the problem supplied with numerical examples. Mishra (2012) explored the inventory model for time dependent holding cost and deterioration with salvage value where shortages are allowed. Padmanavan and Vrat (1995) presented an EOQ inventory model for perishable items with a stock dependent selling rate. They assumed that the selling rate is a function of the current inventory level and the rate of deterioration is taken to be constant. The most recent work found in the literature is that of Hariga (1996) who extended his earlier work by assuming a time-varying demand over a finite planning horizon. Goyal et al. (2001) and Shah (2000) explored the inventory models for deteriorating items. Pattnaik (2010, 2011) studied profit maximization entropic order quantity model for deteriorated items with stock dependent demand where instant deterioration and post deterioration cash discounts respectively are allowed for acquiring more profit. Pattnaik (2011) developed an entropic order quantity model for deteriorating items where cash discounts are allowed but Pattnaik (2011) modified again to obtain the decision parameters for perishable items where instant deterioration discount is allowed in EOQ model. Pattnaik (2012) introduced a non linear profit maximization entropic order quantity model for deteriorating items with stock dependent demand rate. Pattnaik (2012) derived an EOQ model for perishable items with constant demand and instant deterioration.

Furthermore, retailer promotional activity has become more and more common in today's business world. For example, Wall Mart and Costco often try to stimulate demand for specific types of electric equipment by offering price discounts; clothiers Baleno and NET make shelf space for specific clothes items available for longer periods; McDonald's and Burger King often use coupons to attract consumers. Other promotional strategies include free goods, advertising, and displays and so on. The promotion policy is very important for the retailer. How much promotional effort the retailer makes has a big impact on annual profit. Residual costs may be incurred by too many promotions while too few may result in lower sales revenue. Tsao and Sheen (2008) discussed dynamic pricing, promotion and replenishment policies for a deteriorating item under permissible delay in payment. Salameh et al. (1999) studied an EOQ inventory model in which it assumes that the percentage of on-hand inventory wasted due to deterioration is a key feature of the inventory conditions which govern the item stocked. The effect of deteriorating items on the instantaneous profit maximization replenishment model under promotion is considered in this model. The market demand may increase with the promotion of the product over time when the units lost due to deterioration. In the existing literature about promotion it is assumed that the promotional effort cost is a function of promotion. Tripathy et al. (2012) investigated an optimal EOQ model for deteriorating items with promotional effort cost. Pattnaik (2012) explored the effect of promotion in fuzzy optimal replenishment model with units lost due to deterioration. Hence Pattnaik (2013) developed many instantaneous EOQ models and fuzzy EOQ models which are incorporated with promotional effort cost, fixed ordering cost, variable ordering cost and units lost due to deterioration. This model introduces a modified fuzzy EOQ model in which it assumes that a percentage of the on-hand inventory is wasted due to deterioration. There is hidden cost not account for when modeling inventory cost.

This model establishes and analyzes the fuzzy inventory model under profit maximization which extends the classical economic order quantity (EOQ) model. An efficient FEOQ does more than just reduce cost. It also creates revenue for the retailer and the manufacturer. The evolution of the FEOQ model concept tends toward revenue and demand focused strategic formation and decision making in business operations. Evidence can be found in the increasingly prosperous revenue and yield management practices and the continuous shift away from supply-side cost control to demand-side revenue stimulus.

Table 1. *Summary of the Related Researches*

Author(s) and published Year	Structure of the model	Demand	Demand patterns	Deterioration	Units Lost	Setup Cost	Promotional effort cost	Planning	Model
Hariga (1994)	Crisp (EOQ)	Time	Non-stationary	Yes	No	Constant	No	Finite	Cost
Vujosevic et al. (1996)	Fuzzy (EOQ)	Constant, Deterministic	Constant	No	No	Constant	No	Infinite	Profit
Salameh et al. (1999)	Crisp (EOQ)	Constant, Deterministic	Constant	Yes	Yes	Constant	No	Finite	Profit
Pattnaik (2009)	Crisp (EnOQ)	Constant, Deterministic	Constant	Yes (Instant)	No	Constant	No	Finite	Profit
Pattnaik (2011)	Crisp (EOQ)	Constant, Deterministic	Constant	Yes (Instant)	No	Constant	No	Finite	Profit
Tsao et al. (2008)	Crisp (EOQ)	Time and Price	Linear and decreasing	Yes	No	Constant	Yes	Finite	Profit
Tripathy et al. (2009)	Fuzzy (FEOQ)	Constant, Deterministic	Constant	No	No	Constant	No	Finite	Cost
Present Paper (2015)	Fuzzy (FEOQ)	Constant, Deterministic	Constant	Yes (Wasting)	No	Variable	Yes	Finite	Profit

All mentioned above inventory literatures with deterioration has the basic assumption that the retailer owns a storage room with optimal order quantity. In recent years, companies have started to recognize that a tradeoff exists between product varieties in terms of quality of the product for running in the market smoothly. In the absence of a proper quantitative model to measure the effect of product quality of the product, these companies have mainly relied on qualitative judgment. The problem consists of the optimization of fuzzy EOQ model, taking into account the conflicting payoffs of the different decision makers involved in the process. Numerical experiment is carried out to analyze the magnitude of the approximation error. A policy iteration algorithm is designed and optimum solution is obtained through LINGO 13.0 version software. . Finally, sensitivity analyses of the optimal solution with respect to the major parameters are also studied to draw the managerial insights. In order to make the comparisons equitable a particular evaluation function based on promotion is suggested. In this model, replenishment decision under none wasting the percentage of on-hand inventory due to deterioration are adjusted arbitrarily upward or downward for profit maximization model in response to the change in market demand within the finite planning horizon with dynamic setup cost with promotional effort cost. The objective of this model is to determine optimal replenishment quantities in an instantaneous replenishment profit maximization model. However, adding of both promotional effort and dynamic ordering cost in fuzzy model might lead to super gain for the retailer. The major assumptions used in the above research articles are summarized in Table 1.

The remainder of the model is organized as follows. In section 2 notations and assumptions are provided for the development of the model. The mathematical formulation is developed in section 3. Section 4 develops the fuzzy model. In section 5, the solution procedure is given. In section 6, an illustrative numerical analysis is given to illustrate the procedure of solving the proposed model. The sensitivity analysis is carried out in section 7 to observe the changes in the optimal solution. Finally section 8 explains the summary and concluding remarks of the FEOQ model.

2. Assumptions and Notations

r Consumption rate,

t_c Cycle length,

h Holding cost of one unit for one unit of time,

HC (q, ρ) Holding cost per cycle,

K Setup cost per cycle,

c Purchasing cost per unit,

P_s Selling Price per unit,

α Percentage of on-hand inventory that is lost due to deterioration,

q Order quantity,

q** Modified economic ordering / production quantity (FEOQ/FEPQ),

q* Traditional economic ordering quantity (EOQ),

ρ The promotional effort per cycle

PE (ρ) The promotional effort cost, PE (ρ) = K_1 (ρ-1)$^2 r^{\alpha_1}$, where K_1>0 and α_1 is a constant

φ(t) On-hand inventory level at time t,

π_1(q, ρ) Net profit per unit of producing q units per cycle in crisp strategy,

π (q, ρ) Average profit per unit of producing q units per cycle in crisp strategy,

$\tilde{\pi}_1(q,\rho)$ The net profit per unit per cycle in fuzzy decision space,

$\tilde{\pi}$ (q, ρ) The average profit per unit per cycle in fuzzy decision space,

$\tilde{h}$ Fuzzy holding cost per unit,

$\tilde{K}$ Fuzzy setup cost per cycle.

3. Mathematical Model

Denote φ(t) as the on-hand inventory level at time t. During a change in time from point t to t+dt, where t + dt > t, the on-hand inventory drops from φ(t) to φ(t+dt). Then φ(t+dt) is given as:

$$\varphi(t+dt) = \varphi(t) - r\,\rho\, dt - \alpha\,\varphi(t)\, dt$$

φ(t+dt) can be re-written as: $\frac{\varphi(t+dt)-\varphi(t)}{dt} = -r\rho - \alpha\varphi(t)$ and dt → 0, the above equation reduces to: $\frac{d\varphi(t)}{dt} + \alpha\varphi(t) + r\rho = 0$

It is a differential equation, solution is

$$\varphi(t) = \frac{-r\rho}{\alpha} + \left(q + \frac{r\rho}{\alpha}\right) \times e^{-\alpha t}$$

Where q is the order quantity which is instantaneously replenished at the beginning of each cycle of length t_c units of time. The stock is replenished by q units each time these units are totally depleted as a result of outside demand and deterioration. Behavior of the inventory level for the above model is illustrated in Fig. 1. The cycle length, t_c, is determined by first substituting t_c into equation φ(t) and then setting it equal to zero to get: $t_c = \frac{1}{\alpha}\ln\left(\frac{\alpha q + r\rho}{r\rho}\right)$

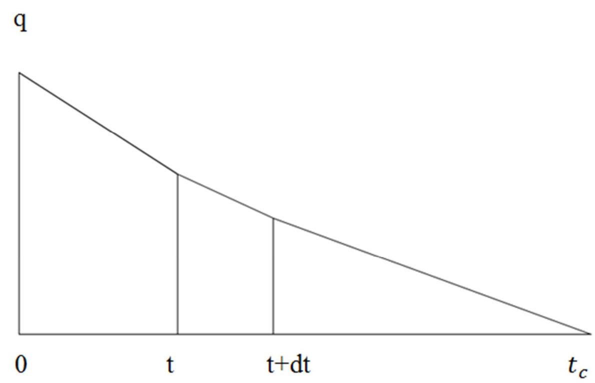

Fig. 1. *Behavior of the Inventory over a Cycle for a Deteriorating Item*

Equation φ(t) and t_c are used to develop the mathematical model. It is worthy to mention that as α approaches to zero, t_c approaches to $\frac{q}{r\rho}$. The total cost per cycle, TC (q, ρ), is the sum of the variable ordering cost and purchasing cost per cycle, $Kq^{(\gamma-1)} + cq$, the holding cost per cycle, HC (q, ρ), and the promotional effort cost per cycle, PE (ρ). HC (q, ρ) is obtained from equation φ(t) as:

$$HC(q,\rho) = \int_0^{tc} h\varphi(t)dt = h\int_0^{\frac{1}{\alpha}ln\left(\frac{\alpha q + r\rho}{r\rho}\right)} \left[-\frac{r\rho}{\alpha} + \left(q + \frac{r\rho}{\alpha}\right) \times e^{-\alpha t}\right] dt$$

$$= h \times \left[\frac{q}{\alpha} - \frac{r\rho}{\alpha^2} ln\left(\frac{\alpha q + r\rho}{r\rho}\right)\right]$$

$$\mathrm{PE}(\rho) = K_1(\rho-1)^2 r^{\alpha_1}$$

$$\mathrm{TC} = \mathrm{OC}+\mathrm{PC}+\mathrm{HC}+\mathrm{PE}$$

$$\mathrm{TC}\ (q, \rho) = Kq^{(\gamma-1)} + cq + h \times \left[\frac{q}{\alpha} - \frac{r\rho}{\alpha^2} ln\left(\frac{\alpha q + r\rho}{r\rho}\right)\right] + K_1(\rho-1)^2 r^{\alpha_1}$$

The total cost per unit of time, TCU (q,ρ), is given by dividing equation TC (q,ρ) by equation t_c to give:

$$TCU(q,\rho) = \left[Kq^{(\gamma-1)} + cq + h \times \left[\frac{q}{\alpha} - \frac{r\rho}{\alpha^2} ln\left(\frac{\alpha q + r\rho}{r\rho}\right)\right] + K_1(\rho-1)^2 r^{\alpha_1}\right] \times \left[\frac{1}{\alpha}\ln\left(\frac{\alpha q + r\rho}{r\rho}\right)\right]^{-1}$$

$$= \frac{Kq^{(\gamma-1)}\alpha + (c\alpha + h)q}{ln\left(1+\frac{\alpha q}{r\rho}\right)} - \frac{hr\rho}{\alpha} + \frac{K_1\alpha(\rho-1)^2 r^{\alpha 1}}{ln\left(1+\frac{\alpha q}{r\rho}\right)}$$

As α approaches zero and ρ = 1 equation $TCU(q,\rho)$ reduces to TCU (q) $= \frac{Kq^{(\gamma-1)}r}{q} + cr + \frac{hq}{2}$. Whose solution is given by the traditional EOQ formula, $q^* = \left[\frac{h}{2Kr(2-\gamma)}\right]^{1/\gamma-3}$.

The total profit per cycle with α approaching to zero only is $\pi_1(q,\rho)$. $\pi_1(q,\rho) = q \times P_s - \mathrm{TC}\ (q,\rho) = qP_s - Kq^{(\gamma-1)} - cq - \frac{hq^2}{2r\rho} - K_1(\rho-1)^2 r^{\alpha 1}$

TC (q, ρ) the total cost per cycle, are calculated from equation TC (q, ρ). Whose solution is given by the traditional EOQ formula, $q^* = \left[\frac{h}{2Kr\rho(2-\gamma)}\right]^{1/\gamma-3}$. The average profit π(q,ρ) per unit time is obtained by dividing t_c in $\pi_1(q, \rho)$. Hence the profit maximization problem is

Maximize π_1 (q,ρ)

$$\forall\ q > 0,\ \rho > 0$$

$$\pi_1(q,\rho) = F_1(q,\rho) + F_2(q,\rho)h + F_3(q,\rho)K$$

Where,

$$F_1(q, \rho) = (q \times P_s) - cq - K_1(\rho-1)^2 r^{\alpha 1},$$

$$F_2(q, \rho) = -\left[\frac{q^2}{2r\rho}\right], \text{ and } F_3(q, \rho) = -q^{(\gamma-1)}$$

4. Fuzzy Mathematical Model

The holding cost and ordering cost are replaced by fuzzy numbers $\tilde{h}$ and $\tilde{K}$ respectively. By expressing $\tilde{h}$ and $\tilde{K}$ as the normal triangular fuzzy numbers *(h_1, h_0, h_2)* and *(K_1, K_0, K_2)*, where, $h_1 = h - \Delta_1$, $h_0 = h$, $h_2 = h + \Delta_2$, $K_1 = K - \Delta_3$, $K_0 = K$, $K_2 = K + \Delta_4$ such that $0 < \Delta_1 < h$, $0 < \Delta_2$, $0 < \Delta_3 < K$, $0 < \Delta_4, \Delta_1, \Delta_2, \Delta_3$ and Δ_4 are determined by the decision maker based on the uncertainty of the problem.

The membership function of fuzzy holding cost and fuzzy ordering cost are considered as:

$$\mu_{\tilde{h}}(h) = \begin{cases} \frac{h-h_1}{h_0-h_1}, & h_1 \le h \le h_0 \\ \frac{h_2-h}{h_2-h_0}, & h_0 \le h \le h_2 \\ 0, & otherwise \end{cases}$$

$$\mu_{\tilde{k}}(K) = \begin{cases} \frac{K-K_1}{K_0-K_1}, & K_1 \le K \le K_0 \\ \frac{K_2-K}{K_2-K_0}, & K_0 \le K \le K_2 \\ 0, & \text{otherwise} \end{cases}$$

Then the centroid for $\tilde{h}$ and $\tilde{K}$ are given by $M_{\tilde{h}} = \frac{h_1 + h_0 + h_2}{3} = h + \frac{\Delta_2 - \Delta_1}{3}$ and $M_{\tilde{k}} = \frac{K_1 + K_0 + K_2}{3} = K + \frac{\Delta_4 - \Delta_3}{3}$ respectively.

For fixed values of q and ρ, let $\pi_1(h, K) = F_1(q,\rho) + F_2(q,\rho)h + F_3(q,\rho)K = y$

Let $h = \frac{y - F_1 - F_3K}{F_2}$, $\frac{\Delta_2 - \Delta_1}{3} = \psi_1$ and $\frac{\Delta_4 - \Delta_3}{3} = \psi_2$

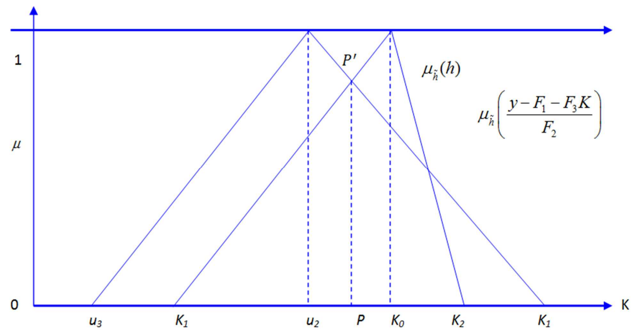

Fig. 2. Defuzzification by using Centroid Method

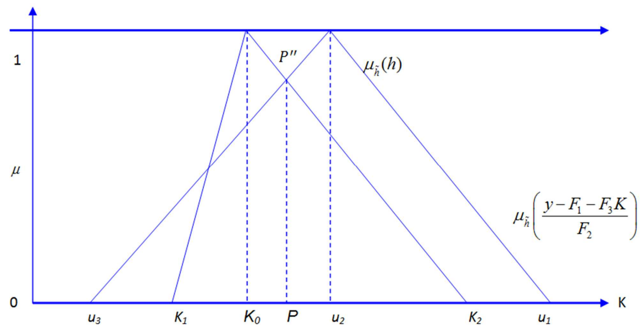

Fig. 3. Defuzzification by using Centroid Method

By extension principle the membership function of the fuzzy profit function is given by

$$\mu_{\pi_1(\tilde{h},\tilde{k})}(y) = \sup_{(h,k)\in\pi_1^{-1}(y)} \{\mu_{\tilde{h}}(h)\wedge\mu_{\tilde{k}}(K)\}$$

$$= \sup_{k_1\le k\le k_2} \left\{\mu_{\tilde{h}}\left(\frac{y-F_1-F_3K}{F_2}\right)\wedge\mu_{\tilde{k}}(K)\right\}$$

Now,

$$\mu_{\tilde{h}}\left(\frac{y-F_1-F_3K}{F_2}\right)=\begin{cases}\dfrac{y-F_1-F_2h_1-F_3K}{F_2(h_0-h_1)}, & u_2\le K\le u_1\\ \dfrac{F_1+F_2h_2+F_3K-y}{F_2(h_2-h_0)}, & u_3\le K\le u_2\\ 0, & \text{otherwise}\end{cases}$$

where, $u_1=\frac{y-F_1-F_2h_1}{F_3}$, $u_2=\frac{y-F_1-F_2h_0}{F_3}$ and $u_3=\frac{y-F_1-F_2h_2}{F_3}$

Fig. 2 exhibits the graph of $\mu_{\tilde{h}}\left(\frac{y-F_1-F_3K}{F_2}\right)$ and $\mu_{\tilde{h}}(h)$ when $u_2\le K$ and $K\le u_1$ then $y\le F_1+F_2h_0+F_3K_0$ and $y\ge F_1+F_2h_1+F_3K_1$. It is clear that for every $y\in[F_1+F_2h_1+F_3K_1,\ F_1+F_2h_0+F_3K_0],\ \mu_y(y)=PP'$. From the $\mu_{\tilde{h}}(h)$ and $\mu_{\tilde{h}}\left(\frac{y-F_1-F_3K}{F_2}\right)$ the value of PP' may be found by solving the following equation:

$$\frac{K-K_1}{K_0-K_1}=\frac{y-F_1-F_2h_1-F_3K}{F_2(h_0-h_1)} \text{ or}$$

$$K=\frac{(y-F_1-F_2h_1)(K_0-K_1)+F_2K_1(h_0-h_1)}{F_2(h_0-h_1)+F_3(K_0-K_1)}$$

Therefore, $PP'=\frac{K-K_1}{K_0-K_1}=\frac{y-F_1-F_2h_1-F_3K}{F_2(h_0-h_1)+F_3(K_0-K_1)}=\mu_1(y)$, (say).

Fig. 3 exhibits the graph of $\mu_{\tilde{h}}\left(\frac{y-F_1-F_3K}{F_2}\right)$ and $\mu_{\tilde{h}}(h)$ when $u_3\le K$ and $K\le u_2$ then $y\le F_1+F_2h_2+F_3K_2$ and $y\ge F_1+F_2h_0+F_3K_0$. It is evident that for every $y\in[F_1+F_2h_0+F_3k_0,\ F_1+F_2h_2+F_3K_2],\ \mu_{\tilde{y}}(y)=PP''$. From the $\mu_{\tilde{h}}(h)$ and $\mu_{\tilde{h}}\left(\frac{y-F_1-F_3K}{F_2}\right)$, the value of PP'' may be found by solving the following equation:

$$\frac{K_2-K}{K_2-K_0}=\frac{F_1+F_2h_2+F_3K-y}{F_2(h_2-h_0)} \text{ or,}$$

$$K=\frac{F_2K_2(h_2-h_0)-(F_1+F_2h_2-y)(K_2-K_0)}{F_2(h_2-h_0)+F_3(K_2-K_0)}$$

Therefore, $PP''=\frac{K_2-K}{K_2-K_0}=\frac{F_1+F_2h_2+F_3K_2-y}{F_2(h_2-h_0)+F_3(K_2-K_0)}=\mu_2(y)$, (say).

Thus the membership function for fuzzy total profit is given by

$$\mu_{\pi_1(\tilde{h},K)}(y)=\begin{cases}\mu_1(y); & F_1+F_2h_1+F_3K_1\le y\le F_1+F_2h_0+F_3K_0\\ \mu_2(y); & F_1+F_2h_0+F_3K_0\le y\le F_1+F_2h_2+F_3K_2\\ 0\ ; & \text{otherwise}\end{cases}$$

Now, let $P_1=\int_{-\infty}^{\infty}\mu_{\pi_1(\tilde{h},\tilde{k})}(y)dy$ and $R_1=\int_{-\infty}^{\infty}y\mu_{\pi_1(\tilde{h},\tilde{k})}(y)dy$

Hence, the centroid for fuzzy total profit is given by

$$\tilde{\pi}_1(q,\rho)=M_{\tilde{TP}}(q,\rho)=\frac{R_1}{P_1}$$

$$=F_1(q,\rho)+F_2(q,\rho)h+F_3(q,\rho)K\ +\psi_1F_2(q,\rho)+\psi_2F_3(q,\rho)$$

$$\tilde{\pi}_1(q,\rho)=M_{\tilde{TP}}(q,\rho)=F_1+(h+\psi_1)F_2+(K+\psi_2)F_3$$

where, $F_1(q,\rho)$, $F_2(q,\rho)$ and $F_3(q,\rho)$ are given by the equations.

Hence the profit maximization problem is

Maximize $\tilde{\pi}_1(q,\rho)=M_{\tilde{TP}}(q,\rho)\ \forall\, q\ge 0,\ \rho\ge 0$

5. Optimization

The optimal ordering quantity q and promotional effort ρ per cycle can be determined by differentiating equation $\tilde{\pi}_1(q,\rho)$ with respect to q and ρ separately, setting these to zero.

In order to show the uniqueness of the solution in, it is sufficient to show that the net profit function throughout the cycle is jointly concave in terms of ordering quantity q and promotional effort factor ρ. The second partial derivates of equation $\tilde{\pi}_1(q,\rho)$ with respect to q and ρ are strictly negative and the determinant of Hessian matrix is positive. Considering the following propositions:

Proposition 1 The net profit $\tilde{\pi}_1(q,\rho)$ per cycle is concave in q.

Conditions for optimal q

$$\frac{\partial\tilde{\pi}(q,\rho)}{\partial q}=P_s-\left((K+\Psi_2)(\gamma-1)q^{\gamma-2}+c+\frac{(h+\Psi_1)q}{r\rho}\right)=0$$

The second order partial derivative of the net profit per cycle with respect to q can be expressed as:

$$\frac{\partial^2\tilde{\pi}(q,\rho)}{\partial q^2}=-\frac{(h+\Psi_1)}{r\rho}-((K+\Psi_2)(\gamma-1)(\gamma-2)q^{\gamma-3}),$$

Since rρ> 0, $(\gamma-1)(\gamma-2)>0$ and $(h+\Psi_1)>0$

Equation $\frac{\partial^2 \tilde{\pi}_1(q,\rho)}{\partial q^2}$ is negative.

Proposition 2 The net profit $\tilde{\pi}_1$ (q, ρ) per cycle is concave in ρ.

Conditions for optimal ρ

$$\frac{\partial \tilde{\pi}_1(q,\rho)}{\partial \rho} = \left(\frac{(h+\Psi_1)q^2}{2r\rho^2}\right) - 2K_1(\rho-1)r^{\alpha 1} = 0$$

The second order partial derivative of the net profit per cycle with respect to ρ is

$$\frac{\partial^2 \tilde{\pi}_1(q,\rho)}{\partial \rho^2} = -\frac{(h+\Psi_1)q^2}{r\rho^3} - 2K_1 r^{\alpha 1}$$

Since $\left(\frac{(h+\Psi_1)q^2}{r\rho^3}\right)$ >0, $K_1 > 0$, r >0, it is found that $\frac{\partial^2 \tilde{\pi}_1(q,\rho)}{\partial \rho^2}$ is negative.

Propositions 12.4.1 and 12.4.2 show that the second partial derivatives of equation π_1 (q,ρ) with respect to q and ρ separately are strictly negative. The next step is to check that the determinant of the Hessian matrix is positive, i.e.

$$\frac{\partial^2 \tilde{\pi}_1(q,\rho)}{\partial q^2} \times \frac{\partial^2 \tilde{\pi}_1(q,\rho)}{\partial q^2} - \left(\frac{\partial^2 \tilde{\pi}_1(q,\rho)}{\partial q \partial \rho}\right)^2 > 0 =$$

$$\frac{2(h+\Psi_1)}{r\rho} K_1 r^{\alpha_1} + (K+\Psi_2)(\gamma-1)(\gamma-2)q^{\gamma-3}\frac{(h+\Psi_1)q^2}{r\rho^3} + 2K_1 r^{\alpha_1}(K+\Psi_2)(\gamma-1)(\gamma-2)q^{\gamma-3} > 0\,,$$

Since $\left(\frac{\partial^2 \tilde{\pi}_1(q,\rho)}{\partial q^2}\right), \left(\frac{\partial^2 \tilde{\pi}_1(q,\rho)}{\partial \rho^2}\right)$ shown in $\frac{\partial \tilde{\pi}_1(q,\rho)}{\partial q}$ and $\frac{\partial \tilde{\pi}_1(q,\rho)}{\partial \rho}$ and $\frac{\partial^2 \tilde{\pi}_1(q,\rho)}{\partial q \partial \rho} = \frac{\partial^2 \tilde{\pi}_1(q,\rho)}{\partial \rho\, \partial q} = \frac{(h+\Psi_1)q}{r\rho^2}$

The objective is to determine the optimal values of q and ρ to maximize the net profit function. It is very difficult to derive the optimal values of q and ρ, hence unit profit function. There are several methods to cope with constraints optimization problem numerically. But here LINGO 13.0 software is used to derive the optimal values of the decision variables.

6. Numerical Example

Consider an inventory situation where K is Rs. 200 per order, h is Rs. 5 per unit per unit of time, r is 1000 units per unit of time, c is Rs. 100 per unit, the selling price per unit P_s is Rs. 125, γ is 0.5 and α is 0%, $K_1 = 2.0$, $\alpha_1 = 1.0$, $\Delta_1 = 0.002$, $\Delta_2 = 0.02$, $\Delta_3 = 0.002$ and $\Delta_4 = 0.2$. The optimal solution that maximizes equation $\tilde{\pi}_1(q,\rho)$ and q^{**} and ρ^* are determined by using LINGO 13.0 version software and the results are tabulated in Table 2. It indicates the present model incorporated with promotional effort cost and variable ordering cost may draw the better decisions in managerial uncertain space.

Fig. 4 represents the relationship between the order quantity q and dynamic setup cost OC. Fig. 5 represents the three dimensional mesh plot order quantity q, promotional effort factor ρ and net profit per cycle $\tilde{\pi}_1(q,\rho)$. Fig. 6 is the sensitivity plotting of order quantity q, promotional effort factor ρ and net profit per cycle $\tilde{\pi}_1(q,\rho)$.

Table 2. *Optimal Values of the Proposed Model*

Model	Iteration	t^*	q^*	ρ^*	OC	PE	$\tilde{\pi}_1(q,\rho)$	$\tilde{\pi}(q,\rho)$
Fuzzy	100	4.994008	82931.88	16.60628	0.6947244	487111.8	549535.9	110039

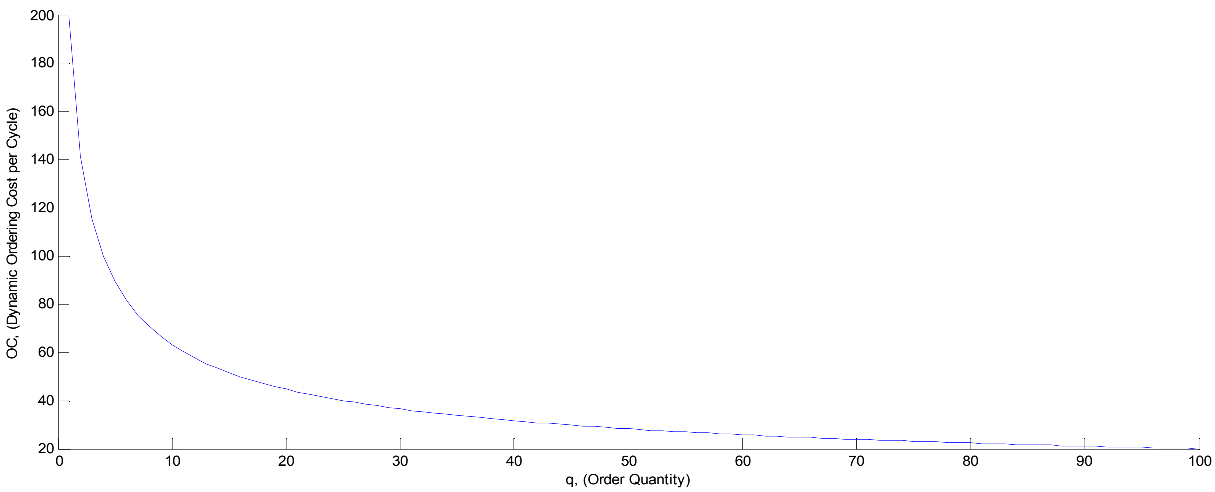

Fig. 4. *Two dimensional plot of Order Quantity, q and Dynamic Ordering Cost, OC*

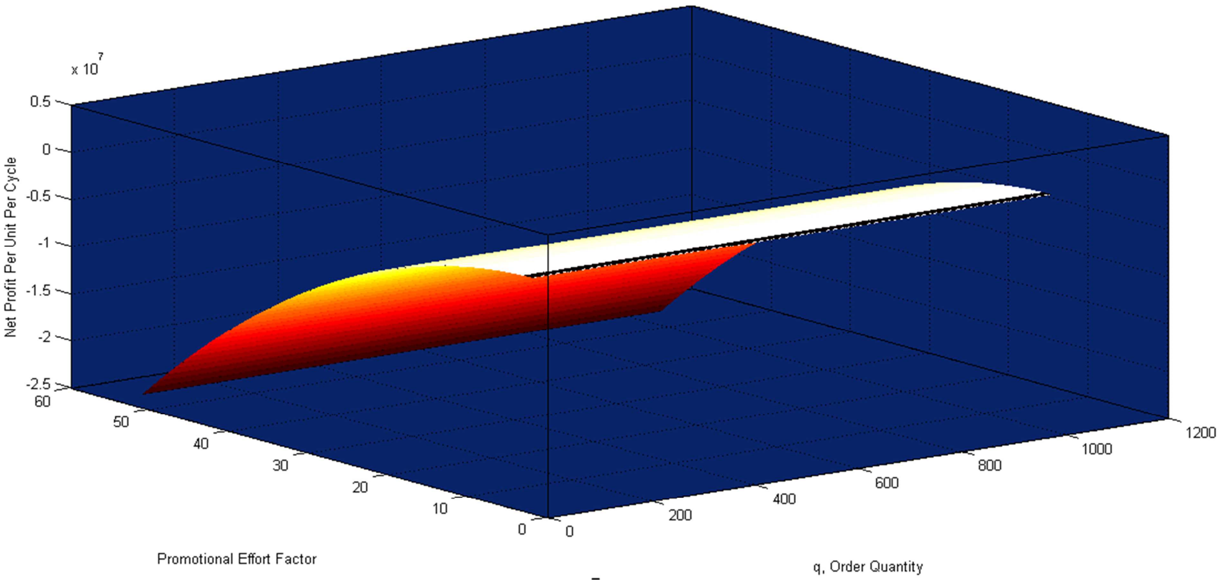

Fig. 5. *Three Dimensional Mesh Plot of Order Quantity q, Promotional Effort Factor ρ and Fuzzy Net Profit per Cycle* $\tilde{\pi}_1(q,\rho)$

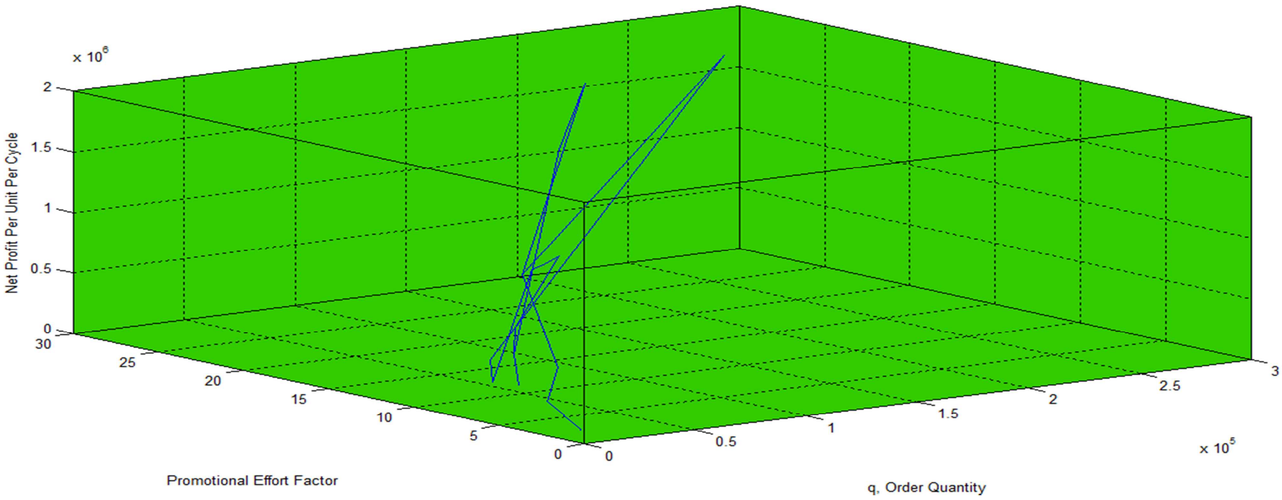

Fig. 6. *Sensitivity Plotting of Order Quantity q, Promotional Effort Factor ρ and Fuzzy Net Profit per Cycle* $\tilde{\pi}_1(q,\rho)$

7. Sensitivity Analysis

It is interesting to investigate the influence of the major parameters $\widetilde{K}$, $\tilde{h}$, r, c, P_s, γ, K_1 and $\alpha_1$on retailer's behaviour. The computational results shown in Table 12.5.4 indicate the following managerial phenomena:

- t_c the replenishment cycle length, q the optimal replenishment quantity, ρ the optimal promotional effort factor, PE promotional effort cost, $\tilde{\pi}_1(q,\rho)$ the optimal net profit per unit per cycle and $\tilde{\pi}$ the optimal average profit per unit per cycle are insensitive to the parameter $\widetilde{K}$ but OC variable setup cost is sensitive to the parameter $\widetilde{K}$.
- t_c the replenishment cycle length, q the optimal replenishment quantity, ρ the optimal promotional effort factor, PE promotional effort cost, OC variable setup cost, $\tilde{\pi}_1(q,\rho)$ the optimal net profit per unit per cycle and $\tilde{\pi}$ the optimal average profit per unit per cycle are sensitive to the parameter $\tilde{h}$.
- t_c the replenishment cycle length and ρ the optimal promotional effort factor and OC variable setup cost is insensitive to the parameter r but q the optimal replenishment quantity, PE promotional effort cost, $\tilde{\pi}_1(q,\rho)$ the optimal net profit per unit per cycle and $\tilde{\pi}$ the optimal average profit per unit per cycle are sensitive to the parameter r.
- t_c the replenishment cycle length, q the optimal replenishment quantity, ρ the optimal promotional effort factor, PE promotional effort cost, OC variable setup cost, $\tilde{\pi}_1(q,\rho)$ the optimal net profit per unit per cycle and $\tilde{\pi}$ the optimal average profit per unit per cycle are sensitive to the parameter c.
- t_c the replenishment cycle length, q the optimal replenishment quantity, ρ the optimal promotional effort factor, PE promotional effort cost, OC variable setup cost, $\tilde{\pi}_1(q,\rho)$ the optimal net profit per unit per cycle and $\tilde{\pi}$ the optimal average profit per unit per cycle are

sensitive to the parameter P_S.

- t_c the replenishment cycle length and ρ the optimal promotional effort factor, q the optimal replenishment quantity, PE promotional effort cost, $\tilde{\pi}_1(q,\rho)$ the optimal net profit per unit per cycle and $\tilde{\pi}$ the optimal average profit per unit per cycle are insensitive to the parameter γ and OC variable setup cost is sensitive to the parameter γ.
- t_c the replenishment cycle length is insensitive to the parameter K_1 but ρ the optimal promotional effort factor, q the optimal replenishment quantity, OC variable setup cost, PE promotional effort cost, $\tilde{\pi}_1(q,\rho)$ the optimal net profit per unit per cycle and $\tilde{\pi}$ the optimal average profit per unit per cycle are sensitive to the parameter K_1.
- t_c the replenishment cycle length is insensitive to the parameter α_1 but ρ the optimal promotional effort factor, q the optimal replenishment quantity, OC variable setup cost, PE promotional effort cost, $\tilde{\pi}_1(q,\rho)$ the optimal net profit per unit per cycle and $\tilde{\pi}$ the optimal average profit per unit per cycle are sensitive with static to the parameter α_1.

Table 3. *Sensitivity Analyses of the Parameters $\tilde{K}$, $\tilde{h}$, r, c, P_s, γ, K_1 and α_1*

Parameter	Value	Iteration	t^*	q^*	ρ^*	OC	PE	$\tilde{\pi}_1(q,\rho)$	$\tilde{\pi}(q,\rho)$
	150	96	4.99401	82931.87	16.60628	0.52110	487111.7	549536	110039.1
$\tilde{K}$	250	115	4.99401	82931.89	16.60628	0.86835	487111.9	549535.7	110039
	500	72	4.99401	82931.94	16.60629	1.73647	487112.3	549534.8	110038.8
	3	79	8.31674	8316.744	8316.744	2.19371	0	103956.6	124499.67
$\tilde{h}$	8	130	3.12266	33594.57	10.75832	1.09154	190449.6	229481.2	73489.02
	10	85	2.49850	22006.41	8.807834	1.34865	121924.6	153153.9	61298.23
	1010	99	4.99401	83761.2	16.60628	0.691277	491982.9	555031.2	111139.4
r	1020	101	4.99401	84590.52	16.60628	0.687871	496854.0	560526.6	112239.8
	1030	100	4.99401	85419.84	16.60628	0.684532	501725.1	566022	113340.2
	103	84	4.39473	57507.23	13.08551	0.834280	292118.9	340459.7	77470.02
c	105	87	3.99521	43899.45	10.98803	0.954869	199521.3	239459.7	59939.8
	108	67	3.39593	27902.17	8.216359	1.197717	104151.7	133015.3	39169.04
	120	82	3.99521	43899.45	10.98803	0.954869	199521.3	239472	59939.8
P_s	128	80	5.59329	115090.4	20.57651	0.589731	766479.6	844784.8	151035.4
	130	120	5.99281	140669.4	23.47304	0.533425	1010075	1099966	183547.6
	0.2	82	4.99401	82931.85	16.60628	0.023238	487111.5	549536.5	110039.2
γ	0.3	78	4.99401	82931.85	16.60628	0.071221	487111.5	549536.5	110039.2
	0.6	79	4.99401	82931.94	16.60628	2.156178	487112.3	549534.4	110038.7
	3	126	4.99401	56952.61	11.40419	0.838333	324741.4	387165.2	77525.97
K_1	5	13`	4.99401	36169.20	7.242516	1.051971	194845.0	257268.6	51515.44
	10	81	4.99401	20581.64	4.121263	1.394547	97422.82	159846	32007.52
	2	78	4.99401	5072.003	1.015607	2.809206	487.1331	62909.39	12596.84
α_1	3	107	4.99401	4994.142	1.000016	2.831019	0.4871336	62422.75	12499.39
	4	119	4.99401	4994.064	1.0001	2.831042	0.0004871	62422.26	12499.29

8. Conclusion

In this model, it investigates the optimal order quantity which assumes that a percentage of the on-hand inventory is not wasted due to deterioration for variable setup cost characteristic features and the inventory conditions govern the item stocked. This paper provides a useful property for finding the optimal profit and ordering quantity for deteriorated items. A new mathematical model with dynamic setup cost is developed. The utilization of variable setup cost makes the scope of the application broader in fuzzy decision space. Further, a numerical example is presented to illustrate the theoretical results, and some observations are obtained from sensitivity analyses with respect to the major parameters. The FEOQ model in this study is a general framework that considers variable setup cost without wasting the percentage of on-hand inventory due to deterioration simultaneously.

In the future study, it is hoped to further incorporate the proposed models into several situations such as shortages are allowed and the consideration of multi-item problem. Furthermore, it may also take partial backlogging into account when determining the optimal replenishment policy. There are many scopes in extending the present work as a future research work. Parameters and decision variables can be considered random or even fuzzy. Effect of shortage, backlogging inflation etc could be added to the multi-item model. Finally, few additional aspects that the near future are the applying dynamic pricing strategy are intended through a new optimization model and stochastically of the quality of the products and these models can be extended by considering the reliability factor as a decision variable.

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