# New twelfth order J-Halley method for solving nonlinear equations 

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#### Abstract

In a paper, Noor and Noor [Predictor--corrector Halley method for nonlinear equations, Appl. Math. Comput., 188 (2) (2007) 1587--1591] have suggested and analyzed a predictor--corrector method Halley method for solving nonlinear equations. In this paper, we modified this method by using the finite difference scheme, which has a quantic convergence. We have compared this modified Halley method with some other iterative of fifth-orders convergence methods, which shows that this new method is a robust one. Several examples are given to illustrate the efficiency and the performance of this new method.


## Keywords

Halley Method, Jarratt Method, Iterative Methods, Convergence Order, Numerical Examples

## 1. Introduction

In recent years, several iterative type methods have been developed by using the Taylor series, decomposition and quadrature formulae, see [1-11] and the references therein. Using the technique of updating the solution and Taylor series expansion, Noor and Noor [10] have suggested and analyzed a sixth-order predictor--corrector iterative type Halley method for solving the nonlinear equations. Also Kou et al. $[6,7]$ have also suggested a class of fifth-order iterative methods. In the implementation of these methods, one has to evaluate the second derivative of the function, which is a serious drawback of these methods. To overcome these drawbacks, we modify the predictor--corrector Halley method by replacing the second derivatives of the function $f$ by its finite difference scheme. We prove that the new modified predictor--corrector method is of fifth-order convergence. We also present the comparison of the new method with the methods of Kou et al. [6, 7]. In passing, we would like to point out the results presented by Kou et al. $[6,7]$ are
incorrect. We also rectify this error.
Several examples are given to illustrate the efficiency and robustness of the new proposed method.

## 2. Iterative Methods

The Jarratt's fourth-order method [8] which improves the order of convergence is defined by

Algorithm 1

$$
\begin{gather*}
y_{n}=x_{n}-\frac{2 f\left(x_{n}\right)}{3 f^{\prime}\left(x_{n}\right)}, f^{\prime}\left(x_{n}\right) \neq 0  \tag{1.1}\\
x_{n+1}=x_{n}-J f \frac{f\left(x_{n}\right)}{f^{\prime}\left(x_{n}\right)} \tag{1.2}
\end{gather*}
$$

where

$$
\begin{equation*}
J f=\frac{3 f^{\prime}\left(y_{n}\right)+f^{\prime}\left(x_{n}\right)}{6 f^{\prime}\left(y_{n}\right)-2 f^{\prime}\left(x_{n}\right)} \tag{1.3}
\end{equation*}
$$

Recently, Kou et al. [5] considered the following two-step iteration scheme

Algorithm 2

$$
\begin{gather*}
y_{n}=x_{n}-J f \frac{f\left(x_{n}\right)}{f^{\prime}\left(x_{n}\right)}, f^{\prime}\left(x_{n}\right) \neq 0  \tag{1.4}\\
x_{n+1}=y_{n}-\frac{f\left(y_{n}\right)}{f^{\prime}\left(y_{n}\right)} \tag{1.5}
\end{gather*}
$$

where

$$
\begin{equation*}
J f=\frac{3 f^{\prime}\left(y_{n}\right)+f^{\prime}\left(x_{n}\right)}{6 f^{\prime}\left(y_{n}\right)-2 f^{\prime}\left(x_{n}\right)} \tag{1.6}
\end{equation*}
$$

We now state some fifth-order iterative methods which have been suggested by Noor and Noor [9] and Kou et al. $[6,7]$ using quite different techniques.

Algorithm 3

$$
\begin{align*}
& y_{n}=x_{n}-\frac{2 f\left(x_{n}\right) f^{\prime}\left(x_{n}\right)}{2 f^{\prime 2}\left(x_{n}\right)-f\left(x_{n}\right) f^{\prime \prime}\left(x_{n}\right)}, f^{\prime}\left(x_{n}\right) \neq 0  \tag{1.7}\\
& x_{n+1}=x_{n}-\frac{2\left[f\left(x_{n}\right)+f\left(y_{n}\right)\right] f^{\prime}\left(x_{n}\right)}{2 f^{\prime 2}\left(y_{n}\right)-\left[f\left(x_{n}\right)+f\left(y_{n}\right)\right] f^{\prime \prime}\left(x_{n}\right)} \tag{1.8}
\end{align*}
$$

which is a two-step Halley method of fifth-order convergent.

In a recent paper Kou et al. [6, 7] have suggested following the iterative methods.

Algorithm 4 (SHM [7]). For a given x0, compute the approximate solution xnp1 by the iterative schemes:

$$
\begin{gather*}
y_{n}=x_{n}-\frac{f\left(x_{n}\right)}{f^{\prime}\left(x_{n}\right)}-\frac{f^{2}\left(x_{n}\right) f^{\prime \prime}\left(x_{n}\right)}{2 f^{3}\left(x_{n}\right)-2 f\left(x_{n}\right) f^{\prime}\left(x_{n}\right) f^{\prime \prime}\left(x_{n}\right)}, f^{\prime}\left(x_{n}\right) \neq 0,  \tag{1.9}\\
x_{n+1}=y_{n}-\frac{f\left(y_{n}\right)}{f^{\prime}\left(x_{n}\right)+\left(y_{n}-x_{n}\right) f^{\prime \prime}\left(x_{n}\right)} . \tag{1.10}
\end{gather*}
$$

Algorithm 5 (ISHM [6]). For a given $x 0$, compute the approximate solution xnp1 by the iterative schemes:

$$
\begin{gather*}
y_{n}=x_{n}-\frac{f\left(x_{n}\right)}{f^{\prime}\left(x_{n}\right)}-\frac{f^{2}\left(x_{n}\right) f^{\prime \prime}\left(x_{n}\right)}{2 f^{3}\left(x_{n}\right)-2 f\left(x_{n}\right) f^{\prime}\left(x_{n}\right) f^{\prime \prime}\left(x_{n}\right)}, f^{\prime}\left(x_{n}\right) \neq 0  \tag{1.11}\\
x_{n+1}=y_{n}-\frac{f\left(y_{n}\right)}{f^{\prime}\left(x_{n}\right)}-\frac{f^{\prime \prime}\left(x_{n}\right) f\left(y_{n}\right)}{2 f^{\prime 3}\left(x_{n}\right)} \tag{1.12}
\end{gather*}
$$

On the basis of above discussion, we propose new Iterative method (FAJT):

Algorithm 6: The iterative technique is given by

$$
\begin{gather*}
y_{n}=x_{n}-\frac{2 f\left(x_{n}\right)}{3 f^{\prime}\left(x_{n}\right)}, f^{\prime}\left(x_{n}\right) \neq 0  \tag{3.1}\\
z_{n}=x_{n}-J f \frac{f\left(x_{n}\right)}{f^{\prime}\left(x_{n}\right)}  \tag{3.2}\\
x_{n+1}=z_{n}-\frac{2 f\left(z_{n}\right) f^{\prime}\left(z_{n}\right)}{2 f^{\prime 2}\left(z_{n}\right)-f\left(z_{n}\right) f^{\prime \prime}\left(z_{n}\right)} \tag{3.3}
\end{gather*}
$$

where

$$
\begin{equation*}
J f=\frac{3 f^{\prime}\left(y_{n}\right)+f^{\prime}\left(x_{n}\right)}{6 f^{\prime}\left(y_{n}\right)-2 f^{\prime}\left(x_{n}\right)} \tag{3.4}
\end{equation*}
$$

## 3. Convergence Analysis of the New Method

In this section, we compute the convergence order of the proposed method (FAJT).

Theorem: Let $\alpha \in I$ be a simple zero of sufficiently differentiable function $f: I \subseteq \mathrm{R} \rightarrow \mathrm{R}$ for an open interval I. If $x_{0}$ is close to $\alpha$, then the three-step algorithm 6 has twelfth order of convergence.

Proof: The iterative technique is given by

$$
\begin{gathered}
y_{n}=x_{n}-\frac{2 f\left(x_{n}\right)}{3 f^{\prime}\left(x_{n}\right)}, f^{\prime}\left(x_{n}\right) \neq 0 \\
z_{n}=x_{n}-J f \frac{f\left(x_{n}\right)}{f^{\prime}\left(x_{n}\right)}, \\
x_{n+1}=z_{n}-\frac{2 f\left(z_{n}\right) f^{\prime}\left(z_{n}\right)}{2 f^{\prime 2}\left(z_{n}\right)-f\left(z_{n}\right) f^{\prime \prime}\left(z_{n}\right)},
\end{gathered}
$$

where

$$
J f=\frac{3 f^{\prime}\left(y_{n}\right)+f^{\prime}\left(x_{n}\right)}{6 f^{\prime}\left(y_{n}\right)-2 f^{\prime}\left(x_{n}\right)}
$$

Let $\alpha$ be a simple zero of $f$. By Taylor's expansion, we have,

$$
\begin{align*}
f\left(x_{n}\right)= & f^{\prime}(\alpha)\left[e_{n}+c_{2} e_{n}^{2}+c_{3} e_{n}^{3}+c_{4} e_{n}^{4}+c_{5} e_{n}^{5}+c_{6} e_{n}^{6}\right. \\
& \left.+c_{7} e_{n}^{7}+c_{8} e_{n}^{8}+c_{9} e_{n}^{9}+c_{10} e_{n}^{10}+O\left(e_{n}^{11}\right)\right]  \tag{3.5}\\
f^{\prime}\left(x_{n}\right)= & f^{\prime}(\alpha)\left[1+2 c_{2} e_{n}+3 c_{3} e_{n}^{2}+4 c_{4} e_{n}^{3}+5 c_{5} e_{n}^{4}+6 c_{6} e_{n}^{5}\right. \\
& \left.+7 c_{7} e_{n}^{6}+8 c_{8} e_{n}^{7}+9 c_{9} e_{n}^{8}+10 c_{10} e_{n}^{9}+O\left(e_{n}^{10}\right)\right] \tag{3.6}
\end{align*}
$$

where

$$
\begin{equation*}
c_{k}=\left(\frac{1}{k!}\right) \frac{f^{(k)}(\alpha)}{f^{\prime}(\alpha)}, k=2,3, \ldots, \text { and } e_{n}=x_{n}-\alpha \tag{3.7}
\end{equation*}
$$

Using (3.1), (3.5) and (3.6), we have

$$
\begin{equation*}
y_{n}=\alpha+\frac{1}{3} e_{n}+\frac{2}{3} c_{2} e_{n}^{2}+\left(\frac{4}{3} c_{3}-\frac{4}{3} c_{2}^{2}\right) e_{n}^{3}+O\left(e_{n}^{4}\right) \tag{3.8}
\end{equation*}
$$

by Taylor's series, we have

$$
\begin{equation*}
f\left(y_{n}\right)=f^{\prime}(\alpha)\left[\frac{1}{3} e_{n}+\frac{7}{9} c_{2} e_{n}^{2}+\left(\frac{37}{27} c_{3}-\frac{8}{9} c_{2}^{2}\right) e_{n}^{3}+O\left(e_{n}^{4}\right)\right. \tag{3.9}
\end{equation*}
$$

and

$$
\begin{align*}
f^{\prime}\left(y_{n}\right)= & f^{\prime}(\alpha)\left[\left(1+\frac{2}{3} c_{2} e_{n}+\left(\frac{4}{3} c_{2}^{2}+\frac{1}{3} c_{3}\right) e_{n}^{2}+\left(4 c_{2} c_{3}\right.\right.\right.  \tag{3.10}\\
& \left.-\frac{8}{3} c_{2}^{3}+\frac{4}{27} 7 c_{4}\right) e_{n}^{3}+O\left(e_{n}^{4}\right)
\end{align*}
$$

Using (3.4), (3.6) and (3.10), we have

$$
\begin{equation*}
J f=1+c_{2} e_{n}+\left(-c_{2}^{2}+2 c_{3}\right) e_{n}^{2}+\left(-2 c_{2} c_{3}+\frac{26}{9} c_{4}\right) e_{n}^{3}+O\left(e_{n}^{4}\right) \tag{3.11}
\end{equation*}
$$

Using (3.2), (3.5), (3.6) and (3.11), we have,

$$
\begin{align*}
z_{n}= & \alpha+\left(\frac{1}{9} c_{4}+c_{2}^{3}-c_{2} c_{3}\right) e_{n}^{4} \\
& +\left(\frac{8}{27} c_{5}+8 c_{3} c_{2}^{2}-\frac{20}{9} c_{2} c_{4}\right.  \tag{3.12}\\
& \left.-2 c_{3}^{2}-4 c_{2}^{4}\right) e_{n}^{5}+O\left(e_{n}^{6}\right)
\end{align*}
$$

by Taylor's series, we have

$$
\begin{align*}
f\left(z_{n}\right)= & \left(\frac{1}{9} c_{4}+c_{2}^{3}-c_{2} c_{3}\right) e_{n}^{4} \\
& +\left(\frac{8}{27} c_{5}+8 c_{3} c_{2}^{2}-\frac{20}{9} c_{2} c_{4}\right.  \tag{3.13}\\
& \left.-2 c_{3}^{2}-4 c_{2}^{4}\right) e_{n}^{5}+O\left(e_{n}^{6}\right)
\end{align*}
$$

and

$$
\begin{align*}
f^{\prime}\left(z_{n}\right)= & \left(\frac{1}{9} c_{4}+c_{2}^{3}-c_{2} c_{3}\right) e_{n}^{4} \\
& +\left(\frac{8}{27} c_{5}+8 c_{3} c_{2}^{2}-\frac{20}{9} c_{2} c_{4}\right.  \tag{3.14}\\
& \left.-2 c_{3}^{2}-4 c_{2}^{4}\right) e_{n}^{5}+O\left(e_{n}^{6}\right)
\end{align*}
$$

Using (3.3), (3.6), (3.12) and (3.14), we get,

$$
\begin{aligned}
x_{n+1}= & \alpha+\left(c_{2}^{11}+\frac{1}{27} c_{4}^{2} c_{2} c_{3}^{2}-\frac{2}{27} c_{4}^{2} c_{3} c_{2}^{3}\right. \\
& -\frac{1}{3} c_{4} c_{3}^{3} c_{2}^{2}+c_{4} c_{3}^{2} c_{2}^{4}-c_{4} c_{3} c_{2}^{6}+\frac{1}{3} c_{4} c_{2}^{8} \\
& -\frac{1}{729} c_{3} c_{4}^{3}+\frac{1}{729} c_{4}^{3} c_{2}^{2}+\frac{1}{27} c_{4}^{2} c_{2}^{5}-4 c_{3} c_{2}^{9} \\
& \left.+6 c_{3}^{2} c_{2}^{7}+c_{3}^{4} c_{2}^{3}-4 c_{3}^{3} c_{2}^{5}\right) e^{12}+O\left(e^{13}\right),
\end{aligned}
$$

implies

$$
\begin{aligned}
e_{n+1}= & \left(c_{2}^{11}+\frac{1}{27} c_{4}^{2} c_{2} c_{3}^{2}-\frac{2}{27} c_{4}^{2} c_{3} c_{2}^{3}-\frac{1}{3} c_{4} c_{3}^{3} c_{2}^{2}\right. \\
& +c_{4} c_{3}^{2} c_{2}^{4}-c_{4} c_{3} c_{2}^{6}+\frac{1}{3} c_{4} c_{2}^{8}-\frac{1}{729} c_{3} c_{4}^{3} \\
& +\frac{1}{729} c_{4}^{3} c_{2}^{2}+\frac{1}{27} c_{4}^{2} c_{2}^{5}-4 c_{3} c_{2}^{9}+6 c_{3}^{2} c_{2}^{7} \\
& \left.+c_{3}^{4} c_{2}^{3}-4 c_{3}^{3} c_{2}^{5}\right) e^{12}+O\left(e^{13}\right) .
\end{aligned}
$$

Thus we observe that the new three-step method (FAJT) has twelfth order convergence.

## 4. Numerical Examples

In this section we now consider some numerical examples to demonstrate the performance of the newly developed iterative method. We compare classical method (NW), Kou et al method (see, [6]) (VCM) and (VSHM), Noor et al. methods (see [12]) (NR1) and (NR2) with the new developed method (FAJT). The comparison is given in Table-2. Some selected functions and their roots are given in Table-1. All the computations for above mentioned methods are performed using software Maple 9, precision 128 digits and $\varepsilon=10^{-15}$ as tolerance and also the following criteria is used for estimating the zero:
(I) $\delta=\left|x_{n+1}-x_{n}\right|<\varepsilon$,
(II) $\left|f\left(x_{n}\right)\right|<\varepsilon$,
(III) Maximum numbers of iterations $=500$.

We used the following examples for comparison:
Table 1. (Some functions and their roots)

| Functions | Roots |
| :--- | :--- |
| $f_{1}=4 x^{4}-4 x^{2}$ | 1 |
| $f_{2}=(x-2)^{23}-1$ | 3 |
| $f_{3}=\exp (x) \cdot \sin (x)+\ln \left(x^{2}+1\right)$ | 3.237562984023 |
| $f_{4}=(x+2) \exp (x)-1$ | -0.442854401002 |
| $f_{5}=x^{3}+4 x^{2}-15$ | 1.631980805566 |
| $f_{6}=\exp \left(x^{2}+7 x-30\right)-1$ | 3 |
| $f_{7}=\exp (1-x)-1$ | 1 |
| $f_{8}=x^{3}-2 x^{2}-5$ | 2.690647448028 |
| $f_{9}=(x-1) \exp (-x)$ | 1 |
| $f_{10}=(1 / x)-1$ | 1 |

Table 2. (Comparison of classical method (NW), Kou's methods (VCM \& (VSHM), Noor's methods (NR1 \& NR2) with the new developed method (FAJT))

|  | Number of iterations | $\mathbf{f}\left(\mathbf{x}_{\mathbf{n}}\right)$ | $\boldsymbol{\delta}=$ Accuracy |
| :--- | :--- | :--- | :--- |
| $\mathrm{f}_{1}, \mathrm{x}_{0}=0.75$ |  |  |  |
| NW | 10 | $7.1 \mathrm{e}-40$ | $5.9 \mathrm{e}-21$ |
| VCM | 33 | 0 | $1.9 \mathrm{e}-42$ |
| VSHM | 8 | $-1.0 \mathrm{e}-127$ | $3.6 \mathrm{e}-25$ |
| NR1 | 5 | $1.8 \mathrm{e}-37$ | $9.5 \mathrm{e}-20$ |
| NR2 | 11 | $3.4 \mathrm{e}-36$ | $4.1 \mathrm{e}-19$ |
| FAJT | 4 | 0 | $3.6 \mathrm{e}-122$ |
| $\mathrm{f}_{2}, \mathrm{x}_{0}=2.9$ |  |  |  |
| NW | 13 | $7.0 \mathrm{e}-44$ | $1.6 \mathrm{e}-23$ |
| VCM | DIVERGE | --- | --- |
| VSHM | DIVERGE | --- | ---- |
| NR1 | 6 | $1.9 \mathrm{e}-31$ | $8.8 \mathrm{e}-19$ |
| NR2 | 20 | $3.1 \mathrm{e}-29$ | $3.5 \mathrm{e}-16$ |
| FAJT | 4 | 0 | $4.7 \mathrm{e}-103$ |


|  | Number of iterations | $\mathrm{f}\left(\mathrm{x}_{\mathrm{n}}\right)$ | $\delta=$ Accuracy |
| :---: | :---: | :---: | :---: |
| $\mathrm{f}_{3}, \mathrm{x}_{0}=2.9$ |  |  |  |
| NW | 7 | -1.1e-51 | $6.6 \mathrm{e}-27$ |
| VCM | DIVERGE | --- | ---- |
| VSHM | 4 | 5.0e-127 | 1.9e-67 |
| NR1 | 4 | -1.0e-9 | $1.2 \mathrm{e}-20$ |
| NR2 | DIVERGE | ---- | ---- |
| FAJT | 3 | $5.0 \mathrm{e}-27$ | 2.5e-69 |
| $\mathrm{f}_{4}, \mathrm{x}_{0}=-.9$ |  |  |  |
| NW | 6 | 3.4e-29 | 5.5e-15 |
| VCM | 4 | $1.0 \mathrm{e}-127$ | $4.8 \mathrm{e}-26$ |
| VSHM | 4 | -7.0e-128 | 5.2e-73 |
| NR1 | 4 | 3.8e-38 | 1.8e-19 |
| NR2 | 45 | $9.6 \mathrm{e}-50$ | $2.8 \mathrm{e}-25$ |
| FAJT | 3 | 0 | $1.0 \mathrm{e}-87$ |
| $\mathrm{f}_{5}, \mathrm{x}_{0}=0.9$ |  |  |  |
| NW | 7 | $6.1 \mathrm{e}-51$ | 2.6e-26 |
| VCM | 6 | 1.0e-126 | 7.7e-43 |
| VSHM | 4 | 0 | $1.5 \mathrm{e}-67$ |
| NR1 | 4 | 5.6e-40 | $8.0 \mathrm{e}-21$ |
| NR2 | 14 | $1.6 \mathrm{e}-30$ | $4.3 \mathrm{e}-18$ |
| FAJT | 3 | 0 | 3.0e-66 |
| $\mathrm{f}_{6}, \mathrm{x}_{0}=2.8$ |  |  |  |
| NW | 17 | 8.2e-33 | 9.8e-18 |
| VCM | DIVERGE | --- | ---- |
| VSHM | DIVERGE | 5.1e-37 | 1.0e-18 |
| NR1 | 8 | $6.9 \mathrm{e}-52$ | 2.8e-27 |
| NR2 | 42 | $1.9 \mathrm{e}-33$ | $4.7 \mathrm{e}-18$ |
| FAJT | 4 | 0 | 5.6e-45 |
| $\mathrm{f}_{7}, \mathrm{x}_{0}=1.1$ |  |  |  |
| NW | 5 | 7.8e-42 | 3.9e-21 |
| VCM | 3 | 0 | 4.3e-39 |
| VSHM | 3 | 0 | 2.2e-42 |
| NR1 | 3 | $2.4 \mathrm{e}-33$ | 7.0e-17 |
| NR2 | 4 | $4.9 \mathrm{e}-37$ | $9.9 \mathrm{e}-19$ |
| FAJT | 2 | 0 | 9.1e-18 |
| $\mathrm{f}_{8}, \mathrm{x}_{0}=2$ |  |  |  |
| NW | 7 | $1.0 \mathrm{e}-37$ | 1.3e-19 |
| VCM | 53 | 0 | 3.7e-29 |
| VSHM | 4 | -1.0e-126 | 2.8e-36 |
| NR1 | 4 | $7.2 \mathrm{e}-38$ | 1.0e-19 |
| NR2 | 9 | 5.8e-51 | 3.1e-26 |
| FAJT | 3 | 0 | 5.6e-52 |
| $\mathrm{f}_{9}, \mathrm{x}_{0}=1$ |  |  |  |
| NW | 1 | 0 | 0 |
| VCM | DIVERGE | --- | ---- |
| VSHM | DIVERGE | --- | ---- |
| NR1 | 1 | 0 | 0 |
| NR2 | DIVERGE | --- | ---- |
| FAJT | 1 | 0 | 0 |
| $\mathrm{f}_{10}, \mathrm{x}_{0}=1.5$ |  |  |  |
| NW | 7 | $2.9 \mathrm{e}-39$ | 5.4e-20 |
| VCM | 4 | 4.6e-105 | $3.0 \mathrm{e}-18$ |
| VSHM | 4 | 0 | $8.3 \mathrm{e}-41$ |
| NR1 | 3 | $1.2 \mathrm{e}-38$ | 1.1e-19 |
| NR2 | DIVERGE | --- | ---- |
| FAJT | 2 | 0 | 0 |

## 5. Conclusion

We observe that our iterative method (FAJT) is comparable with all the methods cited in the Table-1 and
gives better results in terms of less number of iterations / speed and convergence to approximation value very near to the root of the problems. With the help of the technique and idea of this paper one can develop higher-order multi-step iterative methods for solving nonlinear equations, as well as a system of nonlinear equations.

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